Propagation errors analysis of TLE data

R. Wang\textsuperscript{a,\ast}, J. Liu\textsuperscript{a}, Q.M. Zhang\textsuperscript{b}

\textsuperscript{a} Center for Space Science and Applied Research, Chinese Academy of Science, Beijing 100080, China
\textsuperscript{b} State Key Laboratory of Explosion Science and Technology, Beijing Institute of Technology, Beijing 100081, China

Received 31 October 2006; received in revised form 20 November 2008; accepted 21 November 2008

Abstract

Orbit position uncertainty is an important factor for collision avoidance issues. For a single object with high frequency historical data, we can attain its position uncertainty easily. But sometimes data is not enough for errors analysis, orbits need to be classified. In this paper error analysis is made from two-line element sets data (TLEs). The Simplified General Perturbations-4 (SGP4) propagator was used. Statistical errors of debris and R/B are given for lower-altitude orbits which are classified by perigee altitude and eccentricity. The errors results and analysis for SSO (the Sun synchronous orbit) typical orbits are obtained. At last atmospheric drag as a main cause of downrange errors in lower-altitude orbit is analyzed. BSTAR in TLEs is modified to improve prediction precision.

\textsuperscript{\ast} Corresponding author.
\textit{E-mail address: wrl@earth.sepc.ac.cn} (R. Wang).

Keywords: Space debris; Errors analysis; SGP4/SDP4; Lower-altitude orbit; Atmospheric drag; BSTAR

1. Introduction

Debris produced with human activities pollutes the outer space, which pose great hazards to the satellite in orbit. Now three in-orbit collision events have happened and been verified. In order to avoid the catastrophic events, orbit forecasting is necessary. Positions of space objects were forecasted according to the historical orbit data, their intersection relationships were obtained in order to avoid the possible collision events. Due to data errors and forecast errors, the intersection relationships errors inevitably exist. So errors analysis need to be done.

In this paper, TLEs and the corresponding SGP4/SDP4 model are used. And the statistical errors are obtained for orbits of different perigee altitude, eccentricity, inclination in different prediction duration. Classified orbit errors analysis is made first for the orbits below 1000 km perigee altitude, then is for the objects correlated with the typical orbits of Sun synchronous orbit. Atmospheric drag as a main cause of downrange (velocity) errors in low Earth orbits is analyzed. BSTAR is modified in TLEs to improve prediction precision.

2. Data and means

TLEs are used for errors analysis which are “mean” values obtained by removing periodic variations in a particular way. In order to obtain good predictions, these periodic variations must be reconstructed by the prediction model in exactly the same way they were removed by SGP4/SDP4 model suitable for TLEs.

Three months TLE data are picked-up including December 2005, January and February 2006. In order to assure the chosen orbit is not for an object using station-keeping, only debris and R/B are used. SGP4 is used for near-Earth satellites which is an analytical method considering the gravitation and atmospheric drag. SDP4 is used for deep-space satellites considering the gravitational effects of the Moon and the Sun as well as certain sectoral and tesseral Earth harmonics which are of particular importance for half-day and one-day period orbits.

We assumed that the state vector given is accurate and used it as a reference. We propagated the preceding
element set to the epoch of the reference TLE data, and compared the position and its trivariate components: downrange, normal and conormal. We can get trivariate components errors.

3. Classified orbit errors analysis

In this paper, the objects were classified by the perigee altitude and eccentricity. There are four eccentricity classes. The first class is for orbits with eccentricities less than 0.02. The second class is for orbits with eccentricities between 0.02 and 0.2. The third class is for orbit with eccentricities between 0.2 and 0.7. The last is for orbits with eccentricities greater than 0.7. Considering the prediction precision, prediction duration is less than 10 days. Downrange/normal/conormal errors are given for different perigee altitude ranges less than 1000 km.

Here we only give position errors for one-day prediction duration for orbits with perigee altitude below 1000 km. The unit of position errors is kilometer (see Table 1).

For objects in lower-altitude orbits (below about 2000 km), there is an intrinsic uncertainty in the future atmospheric drag, for instance due to random variations in the solar activity. Current methods use the drag perturbation from the recent past to predict drag into the near future (Matney et al., 2004). So errors cannot be negligible.

From Table 1, we can see that for the orbits whose perigee altitude is below 600 km, errors decrease as perigee altitude increases which means that the effects of atmospheric drag are larger than other perturbation factors for orbits below 600 km. For the orbits with perigee altitude below 600 km and eccentricity less than 0.2, errors with eccentricity less than 0.02 are larger than the errors with eccentricity between 0.02 and 0.2 which is because atmospheric drag affects the orbits with eccentricity less than 0.02 more than the orbits with eccentricity between 0.02 and 0.2.

Uncertainty in position prediction is usually described by a trivariate normal distribution. Here position errors are divided into three components: downrange, normal and conormal errors. Here, statistical analysis results are given only for the orbits with 300–400 km perigee altitude. Table 2 gives the proportion relationship among downrange, normal and conormal errors in different prediction period. To simplify the computation, the prediction periods have been selected to be multiples of a full day.

From Table 2, we can verify that for the orbits with 300–400 km perigee altitude, the downrange errors are much larger than other errors. Fig. 1 shows the downrange errors of different prediction time for orbits with perigee altitude is 300–400 km. From Fig. 1, we can get that errors for orbits whose perigee altitude is 300–400 km increase with the prediction duration and the downrange errors are almost a quadratic growth with prediction time. Orbits with different eccentricity have different downrange errors, which is mainly because of the atmosphere drag effects to the whole orbits. For example, for the orbits with the eccentricity less than 0.02, the whole orbits are affected by the atmosphere drag; for the orbits with the eccentric orbits, the atmosphere drag affects the orbits perigee more. The downrange error should be (to first order) approximately proportional to the angular velocity, and the error will be large near perigee and smaller near apogee, causing the uncertainty ellipse to “breathe” as the satellite executes its orbit.

4. Typical orbit errors analysis

There are some typical lower-altitude orbits that are often used. For example, the Sun synchronous orbit (SSO) is a typical orbit of the Earth observation missions.
Many special-use satellites use this kind of orbit. It is a near circular orbit with near 90° inclination and 400–1500 km altitude range.

Fig. 2 shows the inclination and perigee altitude distribution, based on satellite situation report on 15 May, 2006. From Fig. 2, perigee altitude and inclination focus on two districts: one is the orbit with 400–1200 km perigee altitude and 97–101° inclination; the other is the orbit with 1200–1500 km perigee altitude and 100–103° inclination.

Table 3 gives position errors for the orbits with 400–1200 km perigee altitude, 97–101° inclination. $N$ is the sample number, which is the number of TLE sets for all objects in that class. Blank entries mean no enough samples in our database. From Table 3, we can know that most of objects have eccentricities smaller than 0.2. For orbits with eccentricity less than 0.02, errors are the largest for the orbits with 400–500 km perigee altitude which reach 3 km with 1-day prediction duration and decrease to 1 km for orbits with 500–600 km perigee altitude. The propagation errors are small for orbits with greater than 600 km perigee altitude which show no more decrease with increasing altitude. That means that atmospheric drag effects are much smaller for orbits above 600 km altitude. For orbits with 0.02–0.2 eccentricity, errors are not large for the orbits with 500–900 km perigee altitude, but they are larger a bit for the orbits with above 900 km perigee altitude, which should be caused by solar/lunar perturbations.

Table 4 gives position errors for the orbits with 1200–1500 km perigee altitude, 100–103° inclination. $N$ is sample number. From Table 4, these orbits have small eccentricities, most of which are near circular orbits. Errors are decreased by the perigee altitude increase.

5. Atmospheric drag effect to propagation errors and BSTAR modification

From the above analysis, we know that downrange errors are mostly due to atmospheric drag effects in lower-altitude orbits. Atmospheric drag is a kind of non-gravitational effect which is proportional to the area-to-mass ratio of a space object, so satellite attitude has to be considered. Usually, perturbation quantity of atmospheric drag is no more than $10^{-9}$ for the satellites whose area-to-mass ratio is not large compared to the Earth gravitation.

To determine atmospheric drag, SGP4/SDP4 uses a static method to approximate the effect of atmospheric drag on satellite orbits. It models the density of the Earth’s upper atmosphere using the fourth power of the orbital altitude. The drag coefficient (XNDT20) provided in TLE data is determined empirically as a continually updated fit to the changes in revolution per day observed over the long term. Thus, the “XNDT20” drag is the first time derivative of the mean motion (which has the dimension revolutions per day).

For SGP4 and SDP4 users, the mean motion is first recovered from its altered form and the drag effect is obtained from the SGP4 drag term. BSTAR is directly related to the drag term. Since the SGP4/SDP4 model assumes a static atmospheric model and a definition of perigee height that does not depend on the actual argument of perigee, whenever the real drag term changes, the BSTAR term changes by the same factor.

Atmospheric drag is an important factor of position errors for objects in lower-altitude orbit. The downrange errors are considered here which atmospheric drag affects most. BSTAR is a pivotal parameter which represents atmospheric drag in TLE data. Fig. 3 shows the downrange errors and BSTAR changes of 28872 by epoch.

![Fig. 2. Perigee altitude and inclination relationship based on Satellite Situation Report on 15 May, 2006.](image)
From Fig. 3, the trends of BSTAR and downrange errors are coincident which means that BSTAR plays an important role in the downrange errors.

BSTAR in TLEs can be obtained by fitting the orbits in orbit determination over many observations. But it is invariant in prediction process. Errors are small for the short prediction duration, while large for the long prediction duration. So modification of BSTAR is necessary for long-term prediction.

Sometimes, the BSTAR term is missed in TLE data. We can estimate it with published TLE data based on the mean motion \(\dot{X}_N\) and the first time derivative of the mean motion \(\Delta X_{NDT20}\) by

\[
X_{NDT20} = \frac{\Delta X_N}{\Delta T}, \tag{1}
\]

\[
BSTAR = \frac{X_{NDT20}}{X_N}/C2/1.5, \tag{2}
\]
where X0 and XNDT20 have been converted to units of radians and minutes, as in SGP4 and C2 is the value computed during SGP4 initialization.

To verify the formula, historical TLE data of 28872 are used. Fig. 4 is computed and published value comparison of XNDT20 and BSTAR. From Fig. 4, computed and published value of drag term XNDT20 and BSTAR have good coherence. So the above formulas are believable. The computed BSTAR is obtained by computing XNDT20. The BSTAR trend can be known from historical BSTAR data in the prediction duration.

We may get the mean BSTAR in the prediction period by the means of the feed-forward back propagation neural networks (BP neural network) using the historical TLE data. By inputting the historical the Bstar value and the corresponding epoch which were included in the TLE data, the output is the Bstar value at the given prediction epoch. If the TLE data does not include the BSTAR term, we can estimate it by Eq. (2), using the BSTAR mean as the input to propagate the orbit. Object 28872 is analyzed as an example. Table 5 is the chosen data. Data in two epochs are chosen, one epoch (epoch 1) is 05278.9158276 and the other (epoch 2) is 0582.24715354. Prediction will be given from epoch 1 to epoch 2.

From Table 5, BSTAR1 is 0.32753e − 3 in epoch 1 and BSTAR2 is 0.48963e − 3 in epoch 2. BSTAR1 in fact can only represent the drag term before epoch 1. To predict the orbit of object 28872 in epoch 2, we need to find the mean BSTAR from epoch 1 to epoch 2. With the neural network means, we fit the BSTAR before epoch 1 according to the historical data, and then predict the BSTAR from epoch 1 to epoch 2 to get the mean BSTAR 0.40753e − 3. The results are that the downrange error is −110.49268 km with the original BSTAR and 13.36340 km with the modified mean BSTAR. So after BSTAR modification, orbit prediction precision becomes better.

6. Conclusion

Errors analysis is important for collision avoidance problem. Errors for a single object can be obtained directly if data is enough. But in some circumstances space object data are not enough to get statistical error results. In the paper, errors analysis of classified orbits and typical orbit correlated with SSO is obtained by using SGP4/SDP4 model and TLE data. Such results can be directly used for the intersection relationship uncertainty analysis of orbital objects. In this paper we also discuss BSTAR in TLE data and try to modify it. Results show that after BSTAR modification, orbit prediction precision becomes better.

Reference