QUANTITATIVE ANALYSIS OF DESIGN LIMITATIONS FOR SYNTHETIC APERTURE RADIOMETER

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ABSTRACT
Design limitations for synthetic aperture radiometer are quantitatively analyzed. Firstly, the image quality of a small scaled system with a few baselines is analyzed. The image quality is getting poorer due to the truncation in the spatial frequency domain while the number of baselines is getting fewer. This is called the lower limit for the design. Secondly, the design limitation for a large scaled system, where many baselines exist, is analyzed related with system bandwidth. Once the bandwidth of a system is determined, the physical sampling intervals will behavior differently at the upper and lower frequencies which may generate confusion in the retrieved image. This will limit the increasing of spatial resolution while the number of baselines of a system is increasing. This is called the upper limit for the design.

Index Terms—microwave radiometer, aperture synthesis, interferometric radiometer, spatial resolution

1. INTRODUCTION
Microwave synthetic aperture radiometer, also called microwave interferometric radiometers (MIR), is one of the new technologies in passive microwave remote sensing. Its basic principle is to measure with Nyquist samples in the spatial frequency domain (uv-plane) and then to get the image through a Fourier transformation. One sample in the uv-plane is measured by a so called two-element interferometer as two element antennas and coherent receiving channels locate at the two ends of a baseline. Because each element antenna/receiving channel in a sparse antenna array can be combined with any other element to form different baselines, the antenna array can be greatly sparse. Therefore this technology is very suitable for applications in earth observation from space. The first airborne system ESTAR [1] (Electronically Scanned Thinned Array Radiometer) appeared in the late 1980s. And the first space borne system SMOS [2] (Soil Moisture and Ocean Salinity) appeared in 2009. MIR technology was successfully validated and achieving a significant breakthrough in the earth observation area from ESTAR to MIRAS [3]. An important space borne system called GIMS (Geostationary Imaging Microwave Sounder) system with a rotating sparse circular array [4] proposed by NSSC (National Space Science Center) of China, is now under development for future FY-4M meteorology satellite.

Since the image is not taken directly in the spatial domain, but in the spatial frequency domain, the quality and spatial resolution of the image are subject to the constraint of this principle. The quality of the image is affected by truncation error when the maximum baseline or the scale of the system is small. For larger scaled systems, design limitation also exists due to sampling confusion on the uv-plane for the upper and lower limit of the bandwidth. In this paper, quantitative analysis is given to both situations.

2. THE DESIGN LIMIT OF A SMALL SYSTEM
There is a lower design limit for a small-scale MIR system. The MIR system with only a few baselines does not only give a poorer spatial resolution, but also introduces larger errors due to band-limited measurement in the spatial frequency domain. Truncated measurement in the spatial frequency domain results in ripple oscillation in the image after Fourier transformation, and called as the Gibbs phenomenon.

Suppose that the measurement capability for a small scale MIR system is

$$V_w(u) = \begin{cases} V_j(u), & |u| \leq N \\ 0, & \text{else} \end{cases} \quad (1)$$

where $T(\xi)$ is the brightness temperature distribution of the ideal target, $V(u)$ is the corresponding frequency spectrum or the visibility function. $N$ is the cutoff frequency, and $M=2N+1$ is the total number of visibility samples. So equation (1) can be expressed as

$$V_w(u) = V_j(u) W_R(u) \quad (2)$$

where

$$W_R(u) = \begin{cases} 1, & |u| \leq N \\ 0, & \text{else} \end{cases} \quad (3)$$

According to the convolution theorem, the retrieved distribution $T_R(\xi)$ becomes
In order to quantitatively analyze the impact of the truncation errors in the image, simulation has been performed. Suppose a one-dimensional initial distribution in the spatial domain is a normalized square function $T_I(\xi)$, where $\xi \in (-1, 1)$, as shown in Figure 1(a). The visibility function $V_I(u)$ in the spatial frequency domain is obtained by Fourier transform, where $u \in (-u_{\text{max}}, u_{\text{max}})$ and $u_{\text{max}}$ is set large enough to make sure that the square function can be completely represented, as shown in Figure 1(b). However, a small scale MIR system measures only the lower part of the spatial frequency within $u_{\text{max}}$. The retrieved distributions are shown in Figure 2 when the cutoff frequency $N$ is respectively 10, 30 and also $u_{\text{max}}$. The results show that periodic ripple emerges in the retrieved distributions due to truncation in the spatial frequency domain. Although increasing the frequency $M$ would not change the peak of the ripple, it would narrow the width of transition zone and make the effects in the central region much smaller. So the smaller the system is, the poorer quality of the image is, particularly in the central region of the image.

$T_w(\xi) = \frac{1}{2\pi} \left[ T_I(\xi) \ast w_R(\xi) \right]$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} T_I(\xi)w_R(\xi - t)dt$$

where

$w_R(\xi) = \sum_{u=-\infty}^{\infty} W_R(u) e^{i\xi u} = \frac{\sin (2N + 1)\xi}{2\sin \frac{\xi}{2}}$

$$= \frac{\sin M\xi}{\sin \frac{\xi}{2}} \approx \frac{\sin \frac{M\xi}{2}}{\sin \frac{\xi}{2}}$$

In practical applications, the Gibbs phenomenon can be generally reduced by applying the windows such as Hanning, Hamming, Kaiser and Blackman window [5]. Although windowing reduces truncation error in a MIR system, it reduces spatial resolution of the image too. Moreover, Windowing would only obviously reduce the Gibbs phenomenon in the edge of the image rather than in the central region. Consequently, 46.5% regions in the middle of the distribution are selected to further analyze quantitatively the effect resulting from the truncation error. The root mean square (RMS) error between the retrieved distribution of $N=10$ and $N=u_{\text{max}}$ is 0.0080. And RMS error between the retrieved distribution of $N=30$ and $N=u_{\text{max}}$ is 0.0024. This shows that the truncation error has a great impact on a small-scale MIR system. The relation between RMS error and the cutoff frequency or the scale of the system $N$ approximates to be exponential, as shown in Figure 3. When the scale of a MIR system is small enough, the truncation error would rapidly increase. In this situation, even if windowing is used during the image retrieval, the impact of truncation error can’t be eliminated. Therefore, the critical point may be called as the lower design limit for a MIR system. For example, the lower limit of design is $N=20$ in Figure 3.
3. THE DESIGN LIMIT OF A LARGE SYSTEM

Increasing the scale of a MIR system or the number of baselines with a large \( u_{\text{max}} \) not only can improve the quality of the image, but also can improve the spatial resolution of the image. This is the goal of universal application requirements. But in addition to the constraints on project implementation, such as weight, power consumption and installation space, as well as funding constraints, there are still the constraints on design principles and theory in fact. This is because the physical distances between the antenna elements are fixed in practice and the corresponding electrical size varies with the frequency in the bandwidth.

Assuming \( f_0 \) is the central frequency of a MIR system, the bandwidth is \( B=f_0-f_i \), where \( f_0 \) is the upper limit and \( f_i \) is the lower limit of the receiver. So the electrical size of the unit sampling interval at the central frequency is \( d_0=\frac{d}{\lambda_0}=\frac{d f_0}{c} \), the electrical size at the upper limit of frequency is \( d_0=\frac{d f_0}{c} \), and the electrical size at the lower limit of frequency is \( d_0=\frac{d f_i}{c} \), where \( d \) is the physical size of the unit sampling interval, or the shortest baseline and is a constant. \( c \) is light speed. When the bandwidth \( B \) is large and the baseline \( N \) is long, \( N d_0 \) or \( N d_1 \) may lie in different spatial frequency sampling point. However, the whole bandwidth is considered as the single frequency \( f_0 \) in the retrieval algorithm. If there are mixed sampling positions in the system, it will have an impact on the retrieved image and the spatial resolution of the image. The critical points where the confusion begins is

\[
(N-1)d_0 = N d_1 \quad \text{or} \quad N d_0 = (N+1)d_1
\]  

(6)

Substitute \( d_0=\frac{d f_0}{c} \), and \( f_0=f_c+B/2 \), obtain,

\[
(N-1)N = (f_0-B/2)N(f_0+B/2)
\]  

(7)

\[
(N+1)N = (f_0+B/2)N(f_0-B/2)
\]  

(8)

(8)-(7) obtain,

\[
N = f_0 / (B/4 + f_0) \approx f_0 / B
\]  

(9)

Equation (9) is the critical point for a large scale MIR system. When \( N \) is greater than this value, the spatial resolution of the system will be affected due to confusion of the sampling positions within the bandwidth. In practice, this point will be an integer just greater than the value that is given by Equation (9).

In order to verify Equation (9), suppose there is a one-dimensional MIR system, \( f_0=1.4 \text{GHz} \), \( d_{i}=0.5 \), \( B=27 \text{MHz} \). The visibility function can be written as [6]

\[
\tilde{T}_{ij}(\xi, \eta) = \frac{\sqrt{D_i D_j}}{4\pi} T_{ij}(\xi, \eta) - T_{ij}(\xi, \eta) \cdot F_{ni}(\xi, \ eta) \cdot F_{nj}(\xi, \eta) \tag{11}
\]

The modified brightness temperature, \( T_{ij} \) is the brightness temperature of the scene under observation, \( V_{ij} \) is the visibility function. \( f_{ij} \) is the central frequency, \( \Delta \phi = \frac{\pi}{2} d_{ij} \) is the time difference between two antenna elements. \( T_{ij} \) is the electrical size of the receiver (assumed to be the same for all receivers), \( F_{ni} \) and \( F_{nj} \) are the normalized voltage patterns of the two antennas \( i \) and \( j \) with equivalent directivity \( D_i \) and \( D_j \). \( (u, v) \) is the baseline that depends on the antenna position difference: \( (u, v) = (x, y, f_\eta) \), \( \lambda_0 \) and the direction cosines \( (\xi, \eta) = \left(\sin \theta \cos \phi, \cos \theta \sin \phi\right) \) are defined with respect to the \( X \) and \( Y \) axes. Considering the actual measurement in the bandwidth is the integral over the frequency band, Equation (10) becomes

\[
V_{ij}(u, v) = \frac{1}{B} \iint_{\xi \geq \eta \leq \xi + \eta \leq 1} \tilde{T}_{ij}(\xi, \eta) e^{i2\pi fu} d\xi d\eta \tag{12}
\]

Normally, in the case of earth observation, the brightness temperature can be assumed to be uniform in the bandwidth, so Equation (12) becomes

\[
V_{ij}(u, v) = \iint_{\xi \geq \eta \leq \xi + \eta \leq 1} \tilde{T}_{ij}(\xi, \eta) e^{i2\pi fu} d\xi d\eta \tag{13}
\]

In order to examine spatial resolution, suppose the initial brightness temperature distribution \( \tilde{T} = T_{ij} \) is two normalized impulse functions, where \( \theta \in (-60^\circ, 60^\circ) \), as shown in Figure 4. Based on Equation (9), the critical point for the maximum baseline of the system is \( N=52 \). In order to analyze the impact on the retrieved image, four different systems are considered, i.e. \( N=32, 52, 72 \) and 82, as shown in Figure 5. As can be seen from the simulation results, the spatial resolution when \( N = 52 \) is higher than that when \( N=32 \); the spatial resolution when \( N = 72 \) is higher than that when \( N=52 \); the spatial resolution when \( N = 82 \) is very close to that when \( N=72 \).
In order to determine the impact on the retrieved image, for different system scales, the width of the main lobe at the point of the impulse is quantitatively analyzed to judge the obtained spatial resolution. The width of the main lobe, which is defined by the two first nulls of the main lobe for the impulse function in Figure 4, is $1.38^\circ$. The relation between the width of the main lobe and the scales of the system, i.e. the maximum baseline $N$ approximates to be exponential, as shown in Figure 6. The results show that there exists a design limit for a large scale MIR system and it is related with the system bandwidth. When the scale or the maximum baseline $N$ of the system is larger than the upper limit, the ratio of increasing spatial resolution is obviously lower than the ratio of increasing the scale of the system $N$. So the cost to get higher spatial resolution by increasing the scale of a MIR system is not significant after the upper limit given by Equation (9).

![Figure 5: The retrieved distributions with different system scales](image)

![Figure 6: The relation between the spatial resolution and the scale, the maximum baseline $N$, of the MIR system](image)

4. CONCLUSIONS

Design limitations related to the scale of a MIR system are discussed quantitatively. The results show that when the system is small, the quality of the image will be severely affected due to truncation error, which could be considered as the lower limit to the system design. That is to say, the inherent problem about the quality of the image due to small scale can’t be removed for a MIR system. This should be given particular attention when a small-scale ground demonstrator is designed. The results also show that there is an upper limit for large scale MIR system. If the longest baseline is larger than the inverse of the relative bandwidth of the system, the benefit obtained for increasing spatial resolution is getting insignificant compare with the cost and effort of increasing the scale of the system. In other words, the larger the scale for a large MIR system is, the lower the increased spatial resolution is. Therefore, the upper limit should be given particular attention when a large scale MIR system is designed.

5. REFERENCES


