Numerical Simulation of Environment Development of Radiation Belt Particles

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Abstract

Based on the particles dynamics in the radiation belts and the principles of their movement, we establish the particles trajectory simulation method in the radiation belts. On this basis, we first use data of particles in the inner zone for nearly two solar cycles provided by the low altitude weather satellites TIROS/NOAA, and then analyze the change of the flux of high-energy protons in various time and energy scales, as well as the relationship between electron flux angle and electron flux changes in order to obtain the dynamic evolution of computer radiation belt particles numerical simulation. It can be seen from the simulation results that the flux of protons have increased significantly and varied along with both the eleven-year solar cycle and seasons. Besides, the larger the particle energy, the greater the drift velocity, the results and the theoretical predictions are consistent.

Keywords: Radiation Belt, Numerical Simulation, Particles Movement

1. Introduction

Radiation belts, also known as the Van Allen belt, a phenomenon high above the natural environment, are mainly by high-energy charged particles from space captured by the geomagnetic field[1-3]. Man-made radiation, such as high-altitude nuclear explosions release of fission fragments and its β decay electron injected into the radiation belts, captured by the geomagnetic field, will cause the enhancement effect of the natural radiation belts. These natural charged particles are captured, crossing the radiation belts spacecraft performance impact, and therefore, studying the radiation belts of charged particles in motion has important application value.

Movement behavior can be used in the radiation belts of charged particles in the geomagnetic field effect theory to describe the invariant conserved. Coulomb scattering effect of the particles by atmospheric radiation in a low L value tape, will produce the deflection of the energy loss and direction, causing inclination diffusion effect, changing the position and height of the magnetic mirror point, resulting in the destruction of the invariant conserved theory, thus affecting the captured particles exercise behavior and life[4]. According to the structure and spatial distribution, the radiation belts are divided into the inner radiation belt and the outer radiation belt. The inner radiation belt is from the recent capture of charged particles in the region, mainly composed of protons and electrons, as well as a small amount of heavy ions. The spatial extent of the inner radiation belt roughly ranges from L=1.2 to L=2.5, about 600-10000km in the equatorial plane. The energy distribution of the inner radiation belt is from hundreds of kilovolts to hundreds of megabytes volts, and the number of electrons is from tens of kilovolts to 2MeV[5-7]. People immediately recognize the dangers of radiation belts’ space missions in the early era of space science.[8] Radiation belts are most serious radiation environment of the Geo space, and most of the satellite should be run in this environment. The spacecraft in orbit will be met with high-energy particles. High-energy protons and heavy particles will change the status of microelectronic devices on the satellite, so that the spacecraft anomalies or malfunctions, such as single-event upset. It also will make the performance of functional materials. It is a serious threat to the safety of the astronauts and produces radiation threat to human life. Energetic electrons interested satellite housing material for surface and deep charging, and destruction of the satellite surface and inside of the instrument. Another radiation belt energy particle flux increased so that the ring current is increasing rapidly, leading to dramatic changes in the geomagnetic field, which will be in the ground surface sensing a potential difference, called the Earth's surface potential (ESP), and the potential
difference can reach 29V/km. Ground midline between ESP as a voltage source is applied to the Y-shaped connection of the power system to produce geomagnetic induced current (GIC). Compared with the 50Hz AC, GIC may be regarded as DC.[9] When the DC current works as the bias current of the transformer, it will produce the so-called “half-wave saturation”, which will generate so much heat that the transformer damaged even burned. In recent decades, the most striking events are the geomagnetic storm damage to the transmission system in March 1989. This magnetic storm occurs in Quebec City, Canada, a huge damage to the power system, power outages led to 600 million inhabitants of 9 hours, a power loss of $2 \times 10^7$kW, direct economic losses of about $500,000,000$.[10] Therefore, the study and forecasting of the outer radiation belt energetic electrons evolution have important scientific and practical significance.

2. Dynamic processes in the radiation zone

Artificial radiation belts nuclear explosion, nuclear explosion smoke $\beta$ decay particles captured by the geomagnetic field in which the formation of, must study the motion of charged particles in a magnetic field, in order to simulate the generation of artificial radiation belts of charged particles in the Earth even polar subfield movement to meet the Lorentz equation, when considering the radiation belts due to the non-electromagnetic force can be ignored, and kinetic equation of energetic particles in the relativistic [11] is

$$\frac{dm}{dt} = q(E + v \times B) \quad (1)$$

Where $M_0$ is the rest mass of the particles, $q$ is the charge of the particles, and $v$ is the velocity of the particles; $\gamma = 1/\sqrt{1 - (v/c)^2}$ is relativistic factor ($c$ is the speed of light); $E$, and $B$ are respectively the electric field strength and the magnetic field strength, which are related to the electron motion; $t$ represents the time of the moving particles with high energy in the study of radiation; the electric field $E$ becomes minor factor, and it can be ignored in this case, strict solving relativistic particles under the conditions of the equations of motion (1) is more complex, there are certain difficulties, so we need to find a practical method to determine the area of the particle motion. Stömer first proposed the theory of the movement of charged particles in the Earth's dipole field; Rossi summarized the relevant numerical calculation, and utilized this theory to obtain particle motion region. Hypothesis space arbitrary the coordinates of the point as $(r, \lambda, \Phi)$; $r$ is the radial distance, $\lambda$ is the geomagnetic latitude as longitude, $\Phi$, and the equation of motion of the particle can be expressed as

$$\left(\frac{dr}{ds}\right)^2 + r^2 \left(\frac{d\lambda}{ds}\right)^2 = 1 - \left[h_0 \left(\frac{r}{r_s}, \lambda\right)\right]^2 \quad (2)$$

Where $ds$ is determined by

$$\left(ds\right)^2 = \left(dr\right)^2 + r^2 \cos^2 \lambda (d\phi)^2 + r^2 d(\lambda)^2$$

$r_s$ is Stömer radius, and for electron, $r_s = (eM/p)^{0.5}$, $e$ is the electron charge; $M$ is the magnetic moment of the earth magnetic field; $p$ is the momentum of the particles

$$h_0(r/r_s) = -(r/r_s)^2 \cos \lambda - (r/r_s)(2\gamma / \cos \lambda) \quad (3)$$

$$\lambda = 0.5[-(r/r_s) \cos \lambda \sin \chi - (r/r_s) \cos^2 \lambda] \quad (4)$$

$\gamma$ is a constant, $\chi$ is the particle track and the angle between the meridian plane projection.

$$\sin \chi = 0.5[-(r/r_s) \cos \lambda \sin \chi - (r/r_s) \cos^2 \lambda] \quad (5)$$

3. Movement of the particles in the radiation belts

The basic movement of charged particles in the Earth's magnetic field can be broken down into three parts, namely: cyclotron motion around the magnetic field lines; bouncing motion along the magnetic field lines; vertical drift motion of the magnetic field lines (ion westward drift, electronic eastward drift) (Figure 1). The magnetic field of the inner magnetosphere is similar to the dipole field. A capture in the magnetosphere including particles drift along the longitude process, the particles of
the magnetic mirror point high latitudes in both hemispheres to draw two circles, respectively, connected to all of the two circular magnetic segments to form a magnetic shell, known as the drift shell. Drift shell can use a magnetic field strength of the magnetic mirror point $B_m$ and magnetic shell in the equatorial plane of the geocentric distance $L$. The charged particles bound on the drift shell movement exist a very long time, so we call captured particles. In addition to a small amount of heavy ions, 1-100MeV proton and 100KeV-20MeV electron are captured by the Earth's radiation belts.

However, due to the geomagnetic field is not a true dipole field, the motion of charged particles in the actual geomagnetic field also includes particle drift shell splitting and drift loss. In addition, the radiation belts of charged particles constantly collide with atmospheric particles loss. The structure of the actual radiation belts is an inward diffusion equilibrium structure and collision losses.

**Figure 1.** Movement of charged particles in the radiation belts

2D bouncing average Fokker-plank equation is widely used in the description of the magnetic field in the wave-particle interaction. For wave-electron interactions the 2D bouncing average Fokker-plank equation is expressed as follows

$$
\frac{\partial f}{\partial t} = \frac{1}{v} \frac{\partial}{\partial v} \left[ \frac{\partial}{\partial \alpha_e} \left( \frac{\partial f}{\partial \alpha_e} \right) + \frac{\partial f}{\partial \alpha_p} \right] + \frac{1}{v} \frac{\partial}{\partial p} \left[ \frac{\partial}{\partial \alpha_e} \left( \frac{\partial f}{\partial \alpha_e} \right) + \frac{\partial f}{\partial \alpha_p} \right] + \frac{\partial f}{\partial \alpha_e} + \frac{\partial f}{\partial \alpha_p}.
$$

(6)

Here $f$ is the phase space density; $\alpha_e$ equatorial pitch angle electron momentum; $p$ is determined by $m_e$ (electron mass).

$$
\tilde{G} = p^2 T(\alpha_e) \sin \alpha_e \cos \alpha_e
$$

(7)

Which $T \approx 1.30-0.56 \sin \alpha_e$, $<D_{aa}>$, $<D_{pp}>$ and $<D_{ap}>=<D_{pa}>$ respectively represent bouncing pitch angle diffusion coefficient, the average momentum diffusion coefficient and the cross-term diffusion coefficients. These diffusion coefficients under the dipole field are shown below:

$$
< D_{aa} > \equiv \frac{1}{T} \int_0^{\lambda_m} D_{aa} \cos \alpha \cos^2 \alpha \cos^7 \lambda d\lambda
$$

(8)

$$
< D_{pp} > \equiv \frac{1}{T} \int_0^{\lambda_m} D_{pp} \frac{(1 + 3 \sin^3 \lambda)^{\frac{3}{2}}}{\cos \alpha} \cos \lambda d\lambda
$$

(9)

$$
< D_{ap} > \equiv \frac{1}{T} \int_0^{\lambda_m} D_{ap} \frac{(1 + 3 \sin^3 \lambda)^{\frac{3}{2}}}{\cos \alpha_e} \cos^4 \lambda d\lambda
$$

(10)

$\lambda$ is the geomagnetic latitude; $\lambda_m$ is the maximum latitude in the case of the wave exists; $\alpha$ is a Bureau pitch angle; $D_{aa}$, $D_{pp}$ and $D_{ap}$ pitch angle, throwing angular momentum-momentum cross items of local diffusion coefficient. Standard approach to seek diffusion coefficient expressions is to define the intensity of the waves, wherein assuming that the wave is a Gaussian distribution, the wave frequency is $\omega$, the wave normal angle $\theta$ ($\theta$ is the angle of the wave vector and the magnetic field strength). Wave frequency distribution follows:

$$
B^2_\omega = A^2 \exp\left[-(\omega - \omega_m)/\left(\delta \omega\right)^2\right]
$$

(11)
Wherein the lowest cutoff frequency to $\omega_1$, the maximum cutoff frequency of $\omega_2$ in half width $\omega_2$, $A_2$ is a normalization factor, and its expression is:

$$A^2 = \frac{2B_2^2}{\pi \frac{1}{2} \delta \sigma} \left[ \text{erf} \left( \frac{\sigma_2 - \sigma_m}{\delta \sigma} \right) + \text{erf} \left( \frac{\sigma_m - \sigma_1}{\delta \omega} \right) \right]$$

(12)

In summary, according to the typical aggregate acoustic distribution characteristics and the various parameters, we then continue to determine the initial distribution of the electrons and the boundary conditions.

4. Particle simulation method

The FDTD method is a differential principle proposed by K.S.Yee as a basis directly from Maxwell's curl equations summarize the universal law of electromagnetic field, to convert it to the differential equations, and in a certain volume, and in a period of time of continuous electromagnetic field data sampling.[12-14] Therefore, it is the numerical simulation of electromagnetic field problems of the most original, most essential, the most comprehensive, has broad applicability. It is also based on these characteristics of the FDTD algorithm electromagnetic PIC particle simulation mostly as electromagnetic fields using algorithms. Two curl equations of the three-dimensional Cartesian coordinate system can be written as following six scalar equations:

$$\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial z} = - \frac{\partial B_z}{\partial t}$$

(13)

$$\frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial x} = - \frac{\partial B_x}{\partial t}$$

(14)

$$\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial y} = - \frac{\partial B_y}{\partial t}$$

(15)

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = \frac{\partial D_x}{\partial t} + J_x$$

(16)

$$\frac{\partial H_y}{\partial z} - \frac{\partial H_z}{\partial x} = \frac{\partial D_y}{\partial t} + J_y$$

(17)

$$\frac{\partial H_x}{\partial x} - \frac{\partial H_y}{\partial y} = \frac{\partial D_z}{\partial t} + J_z$$

(18)

Using differential principle, we can further obtain the following differential equation:

$$\nabla \cdot \left( \frac{B_x^{n+3/2}}{B_x^{n+1/2,k+1/2}} - \frac{B_x^{n+1/2}}{B_x^{n+1/2,k+1/2}} \right) = \nabla \cdot \left( \frac{E_y^{n+1}}{E_y^{n+1/2,k+1}} - \frac{E_y^{n+1/2}}{E_y^{n+1/2,k+1}} \right)$$

(19)

$$\nabla \cdot \left( \frac{B_y^{n+3/2}}{B_y^{n+1/2,k+1/2}} - \frac{B_y^{n+1/2}}{B_y^{n+1/2,k+1/2}} \right) = \nabla \cdot \left( \frac{E_x^{n+1}}{E_x^{n+1/2,k+1}} - \frac{E_x^{n+1/2}}{E_x^{n+1/2,k+1}} \right)$$

(20)

$$\nabla \cdot \left( \frac{B_z^{n+3/2}}{B_z^{n+1/2,k+1/2}} - \frac{B_z^{n+1/2}}{B_z^{n+1/2,k+1/2}} \right) = \nabla \cdot \left( \frac{E_y^{n+1}}{E_y^{n+1/2,k+1}} - \frac{E_y^{n+1/2}}{E_y^{n+1/2,k+1}} \right)$$

(21)
Where $\Delta t$ is the time step, $\Delta x$, $\Delta y$ and $\Delta z$ in three directions on the grid step, the electromagnetic field component superscript $n$ and $n+1/2$, $n+1$ and $n+3/2$ represent a field the component corresponding to the time step, and the subscript indicates the index of the field component in Yee cell placed.

5. Radiation with computer numerical simulation of the evolution of the particle motion

NASA radiation belt models AE-8 and AP-8 is the average statistics from the 1966 first-generation NASA radiation belt models AE-1 and AP-1 developed from a static model. Data from more than 20 satellites in the early 1960s to the mid-1970s are adopted to construct the AE-8 and AP-8 model data. Model of radiation given with electronics through the quantity and quality of neutron flux energy range were 0.04MeV-7MeV, and 0.1MeV-400MeV, space range were $L=1.14-12$ and $L=1.14-6$, covered in space and energy and so has certain advantages. Electron and proton model of this series can quantitatively give an average particle flux, but it only takes into account the long-term effects of space weather. The model is divided into the syllabus for senior and low years of solar activity, without taking into account the changes in the other time scales. From Figure 2 and Figure 3, we can conclude that small pitch angle electron acceleration effect is weak, large pitch angle acceleration effect is more obvious. An increase in throwing angle electron flux is about 8 times for small pitch angle, while an increase of approximately 50 times occurs to large pitch angle.

Figure 2. Diffusion coefficients at $L = 4.55$, units are per day. The sign of $D_{\alpha\beta}$ is indicated by $\delta$ evaluated at $K_p=3$, and $K_p=5$. 

\[
\begin{align*}
\frac{(D_x)^{n+1/2}_{i,j,k+1/2} - (D_x)^{n}_{i,j,k+1/2}}{\nabla t} &= \frac{(H_y)^{n+1/2}_{i,j+1/2,k+1/2} - (H_y)^{n}_{i,j+1/2,k+1/2}}{\nabla y} - (J_y)^{n+1/2}_{i,j+1/2,k+1/2} \\
\frac{(D_y)^{n+1/2}_{i,j,k+1/2} - (D_y)^{n}_{i,j,k+1/2}}{\nabla t} &= \frac{(H_y)^{n+1/2}_{i,j+1/2,k+1/2} - (H_y)^{n}_{i,j+1/2,k+1/2}}{\nabla y} - (J_y)^{n+1/2}_{i,j+1/2,k+1/2} \\
\frac{(D_z)^{n+1/2}_{i,j,k+1/2} - (D_z)^{n}_{i,j,k+1/2}}{\nabla t} &= \frac{(H_y)^{n+1/2}_{i,j+1/2,k+1/2} - (H_y)^{n}_{i,j+1/2,k+1/2}}{\nabla y} - (J_y)^{n+1/2}_{i,j+1/2,k+1/2}
\end{align*}
\]
Figure 3. Ratio of simulated $f$ without cross diffusion to $f$ with cross diffusion

Distribution in low-altitude proton radiation belts by the effects of geomagnetic secular variation: the center of the geomagnetic dipole field annually 2.5km deviation from the center of the earth (the offset distance of more than 500 km), and the geomagnetic dipole moment over time decreased (0.05%/year attenuation), (see Figure 4). This will make long-term changes in the radiation boundary band slowly drift inward. Apparently, two decades, whether the 16-215MeV energy or the 80-215MeV protons pass volume significantly increased. According to the third definition of the center of the South Atlantic Anomaly, its center drifts about westward of about 5°.

Figure 4. The global proton flux maps at different stages of 215MeV: a) $T_1=1000$, b) $T_2=2000$, c) $T_3=3000$, d) $T_4=4000$

Solar radiation flux density of the 11-year solar cycle changes in low altitude capture periodic variation of the proton and electron fluxes: solar activities go further in the neutral atmosphere corresponding to low solar activity years to be expanded, so radiation with low-altitude boundary due
interaction with atmospheric neutral component and eroded. So the corresponding proton flux, showing changes in the 11-year cycle of solar activity cycle, but the trend is just the opposite.

We analyzed the geomagnetic equator (B/B\textsubscript{min}=1.0) in 1979 to 2001 in energy proton flux changes in the range of 80-215Mev. Each data point in the diagram represents the 80-90 days in the B/B\textsubscript{min} average 1.0 Chu energy in 80-215Mev within the proton flux. The dotted line is that the solar F10.7 flux. Data covering the time range is about two solar activity cycle. From Figure 5 we can initially conclude the following characteristics:

1. The change, during the past two decades, in solar activity (F10.7) with proton flux changes into the opposite trend. Solar maximum when the proton flux is very small, and vice versa. But there is hysteresis that the extremes of the proton flux in solar activity appear after a year or two years.

2. The proton flux exhibits an overall enhancement. It can be clearly seen, after 1990, the maximum value of the proton flux is higher than before proton flux maxima.

**Figure 5.** Proton flux changes in the 80-215Mev at the magnetic equator (B/B\textsubscript{min}=1.0)

In addition to the above long-term changes, proton flux of the 11-year cycles is also showing a trend of seasonal variations, as shown in Figure 6. This may be caused by the seasonal variation of the atmospheric activity. Other seasonal changes of the geomagnetic field activities also may be one of the reasons. For a further explanation to our study, we selected the great years of 1980 and 1990 the two solar activity as the object of study. FIG -5.3 each data point represents the month L=1.3, the energy is greater than 80Mev proton flux average. The following two points can be seen form the figure:

1. The proton flux maxima roughly 1, 5 and September. The minimum value of the proton flux respectively March, July and November.

2. In the last two years of the vernal equinox, the autumnal equinox, respectively, corresponds to the minimum and maximum value of the proton flux.

**Figure 6.** Ratio of simulated without cross diffusion of with cross diffusion, at L = 1.3, with chorus only curves and with both D_{Ll} and chorus, starting at t = 281.5 and at t = 283.4.

Compared to the active solar year, proton flux (P8) with energies greater than 80Mev can be calculated by the NASA AP8 model. Figure 7 represents the high and low solar activity in different B/B\textsubscript{min} energy greater than 80Mev (P8) proton flux with changes in the L value. Preliminary results show the TIROS/NOAA data significantly than the AP8 model predicted to be higher in larger L value. That AP8 model is seriously underestimated larger L value of the proton flux.
Figure 7. Comparison between TIROS/NOAA satellite data and AP8MAX model: a) Solar max, b) Solar min. Solid point TIROS/NOAA satellite data, hollow point the AP8 model calculations of proton flux.

Figures 8 are the global distribution of the proton flux of NASA AP8 MAX model with energy greater than 16MeV and smaller than 80MeV. Compare Figure 8a) and b), and it can be seen the AP8 model seriously underestimated the energy the greater than 16MeV or 80MeV the proton flux. This is also consistent with us to achieve the estimated NASA’s AP8MAX AP8MIN model, which is based on the static model of the sixties and seventies probe data. AP8 did not consider the various factors of the long-term changes of the geomagnetic field.

Figure 8. The global proton flux maps at different stage of 16MeV or 80MeV: a) T1=1000, b) T2=2000, c) T3=3000, T4=4000
6. Conclusion

Based on the data of particles in the inner zone for nearly two solar cycles provided by the low altitude weather satellites TIROS/NOAA, we have analyzed the change of the flux of high-energy protons in various time and energy scales, as well as the relationship between electron flux angle and electron flux changes. The preliminary results indicate that over the past twenty years the flux of protons have increased significantly and varied along with both the eleven-year solar cycle and seasons. Besides, small pitch angle electron acceleration effect is weak, large pitch angle acceleration effect is more obvious. The NASA AP8 models are static, statistical average models based on the data of satellites of the 1970s or earlier. So they are not precise and ignore many problems such as the change of the particles in various time scales. In this paper we compare TIROS/NOAA satellite data and AP8 model predictions. The results show that AP8 underestimated the flux of protons in different energy ranges.

References