Bounce-averaged advection and diffusion coefficients for monochromatic electromagnetic ion cyclotron wave: Comparison between test-particle and quasi-linear models

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1] The electromagnetic ion cyclotron (EMIC) wave has long been suggested to be responsible for the rapid loss of radiation belt relativistic electrons. The test-particle simulations are performed to calculate the bounce-averaged pitch angle advection and diffusion coefficients for parallel-propagating monochromatic EMIC waves. The comparison between test-particle (TP) and quasi-linear (QL) transport coefficients is further made to quantify the influence of nonlinear processes. For typical EMIC waves, four nonlinear physical processes, i.e., the boundary reflection effect, finite perturbation effect, phase bunching and phase trapping, are found to occur sequentially from small to large equatorial pitch angles. The pitch angle averaged finite perturbation effect yields slight differences between the transport coefficients of TP and QL models. The boundary reflection effect and phase bunching produce an average reduction of >80% in the diffusion coefficients but a small change in the corresponding average advection coefficients, tending to lower the loss rate predicted by QL theory. In contrast, the phase trapping causes continuous negative advection toward the loss cone and a minor change in the corresponding diffusion coefficients, tending to increase the loss rate predicted by QL theory. For small amplitude EMIC waves, the transport coefficients grow linearly with the square of wave amplitude. As the amplitude increases, the boundary reflection effect, phase bunching and phase trapping start to occur. Consequently, the TP advection coefficients deviate from the linear growth with the square of wave amplitude, and the TP diffusion coefficients become saturated with the amplitude approaching 1 nT or above. The current results suggest that these nonlinear processes can cause significant deviation of transport coefficients from the prediction of QL theory, which should be taken into account in the future simulations of radiation belt dynamics driven by the EMIC waves.


1. Introduction

[2] The electron radiation belt can exhibit dramatic variability during geomagnetic storms [Friedel et al., 2002].

Typically, there is a rapid dropout of relativistic electron fluxes in the outer radiation belt during the main phase, and a gradual buildup to levels probably exceeding the prestorm value by several orders of magnitude during the recovery phase [Reeves et al., 1998]. Over the past decade, increasing attention has been paid to understanding and forecasting the electron radiation belt evolution due to its great scientific and practical significance (see review by Li and Temerin [2001], Friedel et al. [2002], Millan and Thorne [2007], and Thorne [2010]).

[3] The physical mechanisms controlling radiation belt dynamics can be classified into the adiabatic and non-adiabatic types. The adiabatic transport associated with the storm-time geomagnetic field changes can partially explain the dropout and buildup of electron fluxes during the main phase and recovery phase [McIlwain, 1966; Kim and Chan, 1997; Su et al., 2010b]. In fact, the change rates in storm-time electron fluxes are often observed to be beyond the expected...
values of adiabatic process [e.g., Li et al., 1997; Horne et al., 2005b; Bortnik et al., 2006], indicating the existence of nonadiabatic physical processes, such as various wave-particle interactions [Thorne, 2010]. The ultra low frequency (ULF) waves can drift-resonate with radiation belt electrons [Zong et al., 2007, 2009], and cause radial diffusion [Elkington et al., 1999, 2003] and/or convection [Degeling et al., 2008] violating the third adiabatic invariant. The cyclotron/Landau resonances with chorus, hiss, electromagnetic ion cyclotron (EMIC), magnetosonic, R-X and Z-mode waves have been suggested to be the important in-situ acceleration and/or loss mechanisms violating the first and second adiabatic invariants [Horne and Thorne, 1998; Summers et al., 1998, 2004, 2007a, 2007b; Roth et al., 1999; Summers and Ma, 2000; Albert, 2003, 2005; Horne et al., 2003, 2005a, 2007; Xiao et al., 2007, 2010b, 2012]. The bounce-resonance with magnetosonic and EMIC waves violating the second adiabatic invariant may also potentially contribute to the local acceleration and loss of radiation belt electrons [Roberts and Schulz, 1968; Shprits, 2009]. The nonlinear phase trapping and phase bunching by large amplitude chorus and EMIC waves have been investigated based on the test-particle simulation [Summers and Omura, 2007; Bortnik et al., 2008; Albert and Bortnik, 2009a; Yoon, 2011; Tao et al., 2012] and Hamilton theory [Albert, 2002]. In addition, the non-resonant effect of magnetosonic wave can be facilitated by its low frequency and spatial confinement, as identified by the recent test-particle simulations [Bortnik and Thorne, 2010].

[4] Several global models for electron radiation belt dynamics have been constructed based on the Fokker-Planck-type equations, like Salammbô [Beutier and Bosch, 1995; Varotsou et al., 2005, 2008], RAM [Jordanova and Miyoshi, 2005], RBE [Fok et al., 2008], VERB [Shprits et al., 2009b; Subbotin and Shprits, 2009; Subbotin et al., 2010] and STEERB [Su et al., 2010a, 2010d, 2011b; Xiao et al., 2010a]. All of these kinetic models include some local cyclotron resonant interactions with their corresponding transport coefficients determined by the quasi-linear theory [e.g., Albert, 2003, 2005; Horne et al., 2003; Glauert and Horne, 2005; Summers, 2005; Shprits et al., 2006; Li et al., 2007; Ni et al., 2008; Shprits and Ni, 2009; Xiao et al., 2009; Su et al., 2009a, 2011c]. These models appear to be able to reproduce the observed radiation belt evolution on both the short (days) [Miyoshi et al., 2003; Albert et al., 2009; Su et al., 2011a] and long (months) [Subbotin et al., 2011] timescales.

[5] The incoherent EMIC spectrums are often observed in the space [e.g., Anderson et al., 1992a, 1992b], and the narrow band EMIC packets with rising tones have been reported [Pickett et al., 2010]. Following the previous work [Albert and Bortnik, 2009a], we shall concentrate on the interaction between radiation belt electrons and monochromatic EMIC wave. The EMIC wave is assumed to be parallel-propagating away from equator with angular frequency \( \omega \), wave vector \( k \) and wave-amplitude \( B_\omega \). Its corresponding dispersion relation in a multi-ion plasma is given by [Summers and Thorne, 2003]

\[
\mu^2 = \frac{c^2 k^2}{\omega^2} = 1 - \frac{\omega_{pe}^2}{\omega^2} - \frac{3}{\omega^4} \sum_{j=1}^{3} \frac{\omega_{pj}^2}{\omega^2 (\omega - \Omega_j)}
\]

where subscripts \( j = 1,2,3 \) denote the ion species \( H^+, \) \( He^+ \) and \( O^+ \), respectively; \( \Omega_j \) and \( \omega_{pj} \) are the gyro-frequencies and plasma frequencies for ions; \( \Omega_{pe} \) and \( \omega_{pe} \) are the gyro-frequency and plasma frequency for electrons.

[6] The background geomagnetic field is assumed to be dipolar

\[
B = \frac{B_0}{L} \sqrt{1 + \frac{3 \sin^2 \lambda}{\cos^6 \lambda}},
\]

where \( B_0 = 31200 \text{ nT} \) is the equatorial field magnitude at the Earth’s surface; \( L \) is the magnetic shell; \( \lambda \) is the magnetic latitude. The transformation between magnetic latitude \( \lambda \) and field line length \( s \) is implemented by

\[
\frac{ds}{d\lambda} = R_E \cos \lambda \sqrt{1 + \frac{3 \sin^2 \lambda}{\cos^6 \lambda}}
\]

with the Earth’s radius \( R_E \).

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2.2. Test-Particle Model

The relativistic gyro-averaged equations of motion of test particles can be written as [Albert and Bortnik, 2009a]

\[
\frac{dp_{||}}{dt} = \frac{eB_w}{\gamma m_e} p_{\perp} \sin \eta - \frac{p_{||}^2}{2\gamma m_e B} \frac{\partial B}{\partial s}, \tag{4}
\]

\[
\frac{dp_{\perp}}{dt} = eB_w \left( \omega - \frac{p_{||}}{\gamma m_e} \right) \sin \eta + \frac{p_{||} p_{\perp}}{2\gamma m_e B} \frac{\partial B}{\partial s}, \tag{5}
\]

\[
\frac{d\eta}{dt} = eB_w \left( \omega - \frac{p_{||}}{\gamma m_e} \right) \cos \eta + \frac{k_{\parallel}}{\gamma m_e} - \frac{\Omega_e}{\gamma}, \tag{6}
\]

\[
\frac{ds}{dt} = \frac{p_{||}}{\gamma m_e}. \tag{7}
\]

These equations describe the evolution of parallel momentum \( p_{||} \), perpendicular momentum \( p_{\perp} \), wave-particle phase \( \eta \) and field line length \( s \), respectively, due to both background and wave-related electromagnetic fields. In the absence of waves, they degenerate into the adiabatic equations of bounce motion. When \( d\eta/dt = 0 \) in equation (6), the linear cyclotron resonance condition

\[
\omega = \frac{k_{\parallel}}{\gamma m_e} - \frac{\Omega_e}{\gamma} \tag{8}
\]

can be obtained ignoring the term on the order of \( B_w/B \).

The competition between the adiabatic motion and wave-induced motion is characterized by a dimensionless parameter [Omura et al., 2008; Albert and Bortnik, 2009b]

\[
R = \frac{B}{B_w} \frac{\mu}{\mu^2 - 1} \frac{1}{\beta_{\perp} \omega} \left[ \frac{\gamma \mu}{\omega} \omega \beta^2 + \beta_{||} \right] \frac{1}{\dot{\beta}_{||}} \frac{\gamma \beta^2}{\omega} \frac{\partial B}{\partial s} - \frac{\gamma \beta^2}{2\mu} \frac{\dot{\beta}_{||}}{\partial s}, \tag{9}
\]

with

\[
\beta_{||} = \frac{p_{||}}{\gamma m_e c}, \beta_{\perp} = \frac{p_{\perp}}{\gamma m_e c}. \tag{10}
\]

where all the quantities are evaluated at the resonance point.

The linear behaviors are expected to occur in the region \( R \gg 1 \), while the nonlinear behaviors are expected to arise in the region \( R \ll 1 \). The definition of \( R \) is problematic at the equator, where \( \partial B/\partial s = 0 \) and consequently \( R \rightarrow 0 \) (independent of the wave amplitude). It doesn’t mean that the nonlinear processes could occur for an infinitesimal amplitude wave [Inan et al., 1978].

[11] The current system of differential equations (4)–(7) can be solved by the Bulirsch-Stoer method [Stoer and Bulirsch, 1980]. For arbitrary equatorial pitch angle \( \alpha_{eq} \) and kinetic energy \( E_k \), the trajectories of 24 test electrons are calculated. These test electrons are launched from the equator toward the northern hemisphere, with initial wave-particle phase \( \eta_0 \) uniformly distributed in the range between 0° and 360°. The bounce-averaged test-particle advection and diffusion coefficients in pitch angle are statistically calculated based on the following expressions [Bortnik and Thorne, 2010; Tao et al., 2011; Zheng et al., 2012]

\[
\langle \alpha_{eq}^{TP} \rangle = \left( \frac{\alpha_{eq}^0 - \alpha_{eq}}{\Delta \alpha_{eq}^{TP}} \right), \tag{11}
\]

\[
\langle D_{\alpha_{eq}^{TP}} \rangle = \left( \frac{\alpha_{eq}^0 - \alpha_{eq}}{\Delta \alpha_{eq}^{TP}} \right)^2, \tag{12}
\]

where \( \Delta \alpha_{eq}^{TP} \) is the transit time of test electrons back and forth between the mirror point and equator (about one half of the bounce period); \( \alpha_{eq}^0 \) is the final equatorial pitch angle; the overline \( \overline{\alpha_{eq}} \) represents the averaging over the initial phase.

2.3. Quasi-Linear Model

[12] The quasi-linear bounce-averaged kinetic equations of radiation belt electrons driven by EMIC waves can be written as [Schulz and Lanzerotti, 1974; Lyons and Williams, 1984]

\[
\frac{\partial f}{\partial t} = \frac{1}{\gamma} \frac{e}{\sin \alpha_{eq}} \left( G(\alpha_{eq}) \frac{\partial f}{\partial \alpha_{eq}} \right), \tag{13}
\]

\[
G = T(\alpha_{eq}) \sin \alpha_{eq} \cos \alpha_{eq}, \tag{14}
\]

\[
T(\alpha_{eq}) \approx 1.30 - 0.56 \sin \alpha_{eq}, \tag{15}
\]
where $f$ is the phase space density; $\langle D_{\alpha_{eq}}^{QL} \rangle$ is the quasi-linear bounce-averaged pitch angle diffusion coefficient depending on the properties of wave spectra and background plasma.

Equation (13) can be rewritten into the advection-diffusion form [Schulz and Lanzerotti, 1974]

$$\frac{\partial f}{\partial t} = -\frac{1}{G \alpha_{eq}} \left( G \langle A_{\alpha_{eq}}^{QL} \rangle f \right) + \frac{1}{G \alpha_{eq}^2} \left( G \langle D_{\alpha_{eq}}^{QL} \rangle f \right),$$

where $\langle A_{\alpha_{eq}}^{QL} \rangle$ is the inherent advection coefficient

$$\langle A_{\alpha_{eq}}^{QL} \rangle = \frac{1}{G \alpha_{eq}} \left( G \langle D_{\alpha_{eq}}^{QL} \rangle \right).$$

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[14] For broadband field-aligned EMIC waves, the quasi-linear bounce-averaged pitch angle diffusion coefficient is given by [Summers, 2005; Tao et al., 2011; Xiao et al., 2011]

$$\langle D_{\alpha_{eq}}^{QL} \rangle = \frac{1}{T(\alpha_{eq})} \int_0^{\lambda_0} \cos \alpha \cos \alpha_{eq}^7 \alpha \lambda d\lambda,$$

and

$$D_{\alpha_{eq}}^{QL} = \frac{\pi}{4 \left( \frac{\alpha_{eq}}{\gamma} \right)^2} \sum \frac{W}{B^2} \left( 1 - \frac{\gamma \alpha}{\gamma m_e \omega} \frac{dy}{dx} \right)^2 \left( 1 - \cos \alpha \frac{\gamma m_e \omega}{\gamma m_e \omega} \frac{dy}{dx} \right)^{-1},$$

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Figure 2. (top) Variation of the equatorial pitch angle $\alpha_{eq}$ along latitude $\lambda$; (middle) net change of the equatorial pitch angle $\Delta \alpha_{eq}$ as a function of initial wave-particle phase $\eta_0$; (bottom) variation of the wave-particle phase $\eta$ along latitude $\lambda$. In the middle panels, the dotted lines represent the mean changes, and the dashed lines represent the mean changes plus or minus one standard deviation. In the bottom panels, the vertical dashed lines represent linear resonance locations determined by equation (8). The three columns correspond to $E_k = 2$ MeV electrons with the initial equatorial pitch angles $\alpha_{eq} = 15^\circ$, $45^\circ$ and $55^\circ$, respectively. The color coding represents the cases with different initial wave-particle phase $\eta_0$. The red arrow denotes the phase bunching location.
where $\lambda_m$ is the latitude of mirror point; $D_{\alpha\alpha}^{QL}$ is the local diffusion coefficient in pitch angle $\alpha$; $W$ is the Gauss-type wave spectral intensity with lower limit $w_1$, upper limit $w_2$, central frequency $w_m$ and half width $D_w$; the sum $\sum$ is implemented over all the resonance roots obeying the dispersion relation (3) and the cyclotron resonance condition (8).

[15] Based on the derivations of Albert [2000, 2010], we can obtain the quasi-linear bounce-averaged pitch angle diffusion coefficient for a monochromatic EMIC wave

$$D_{\alpha\alpha}^{QL} = \frac{\pi \Omega^2 \cos \alpha \cos^2 \lambda B^2}{6} \left( 1 - \frac{\gamma m_e \omega}{p \kappa} \cos \alpha \right)^2$$

where all the quantities without subscript “eq” are evaluated at the resonance location.

[16] The expressions (17) and (22) have some inherent singular lines. The equatorial resonance curve (along which $\frac{\partial}{\partial \alpha} \left( \frac{k\parallel}{\gamma m_e} - \frac{\Omega_\parallel}{\gamma} \right) = 0$) is a singular line of $\langle D_{\alpha\alpha}^{QL} \rangle$ and $\langle A_{\alpha\alpha}^{QL} \rangle$. The other singular line of $\langle A_{\alpha\alpha}^{QL} \rangle$ is $\alpha_{eq} = 0$ (where $G = 0$).

3. Numerical Results

3.1. Dependence on Pitch Angle and Energy

[17] The typical amplitudes of EMIC waves range from 1 to 10 nT in the outer radiation belt [Meredith et al., 2003; Fraser et al., 2010], excited by the enhanced anisotropy of ring current protons during geomagnetic storms [Anderson et al., 1996; Horne and Thorne, 1993; Jordanova et al., 2008]. In this subsection, the amplitude of EMIC wave is assumed to be $B_w = 2$ nT.

[18] Figure 1 shows the parameter $R$ for a range of kinetic energies from $E_k = 1$ to 10 MeV, and equatorial pitch angles from $\alpha_{eq} = 0^\circ$ to $90^\circ$. In the resonant range, $R$ decreases rapidly with the increase of pitch angle, and increases gradually with the increase of energy. Clearly, the values of $R$ are close to or less than 1 in most of the resonant regions, demonstrating the wide presence of nonlinear physical processes. Hence, there should be notable differences between the bounce-averaged pitch angle transport coefficients of TP model (including the nonlinear effect) and QL model.

[19] Figure 2 plots three examples of the evolution of equatorial pitch angle $\alpha_{eq}$, the dependence of net pitch angle change $D_{\alpha_{eq}}$ on the initial wave-particle phase $\eta_0$, and the evolution of wave-particle phase $\eta$ for $E_k = 2$ MeV electrons. It is found that the resonance locations and the amplitudes of pitch angle changes are generally consistent with those obtained by Albert and Bornik [2009a]. Three different regimes of cyclotron resonance with EMIC waves are exhibited here. For initial $\alpha_{eq} = 15^\circ$, the actual resonance locations (where $dy/dt = 0$) are close to the linear one determined by equation (8), and the resonance wave-particle phases $\eta$ are almost uniformly distributed between $0^\circ$ and $90^\circ$.
360°. The net pitch angle change \( \Delta \alpha_{eq} \) varies quasi-sinuso-dal with the initial wave-particle phase \( \eta_0 \), indicating that the average motions of this group of electrons are dominated by the diffusive process [Bortnik et al., 2008]. For initial \( \alpha_{eq} = 45° \), most of the electrons’ actual resonance latitudes are below the linear one, with the corresponding resonance wave-particle phases \( \eta \) bunching around 300°. The net pitch angle change \( \Delta \alpha_{eq} \) is nearly deterministic and largely independent of \( \eta_0 \), suggesting that the average motions of this group of electrons are dominated by the advective process [Albert and Bortnik, 2009a]. For initial \( \alpha_{eq} = 55° \), almost one half of the electrons experience 2–3 resonances with the wave-particle phases \( \eta \) trapping in a limited range. The net change in equatorial pitch angle \( \Delta \alpha_{eq} \) has both relatively large mean (in absolute value) and standard deviation, implying that the average motions of this group of electrons can be described by a combination of the advective and diffusive processes [Albert and Bortnik, 2009a].

[20] Figure 3 shows the two-dimensional distributions of bounce-averaged pitch angle advection and diffusion coefficients obtained from the TP and QL models in the \((\alpha_{eq}, E_k)\) space. In contrast to the smooth distributions of QL transport coefficients, TP transport coefficients include a series of strong striations (corresponding to the fishbone-like structures of the transport coefficient profiles in Figure 4). It should be noted that the similar striations can be found in the transport coefficients for monochromatic chorus wave [Albert 2000, Plate 1]. The advection coefficients of TP model have both positive and negative values, whereas those of QL model keep positive in the resonant region. The absolute values of \( A_{\alpha_{eq}}^{TP} \) are less than 2, much smaller than those of \( \langle A_{\alpha_{eq}}^{QL} \rangle \) near the singular lines. The diffusion coefficients of TP model decrease rapidly around the loss cone, significantly deviating from the predication of QL theory.

[21] The difference between the transport coefficients of TP and QL models is clearly shown in Figure 4. Several examples of test electron trajectories are presented in Figures 5, 6, and 7 to explain the discrepancy between TP and QL results.

[22] First, we focus on the transport coefficients of \( E_k = 2.0 \) MeV electrons to study the pitch angle dependence of the differences between TP and QL models.

[23] The diffusion coefficients of QL model are about 5 times (or above) higher than those of TP model around the loss cone, where the linear behaviors are expected (since \( R \gg 1 \)). It is found that a part of test electrons can move down to the lower equatorial pitch angle boundary \( \alpha_{eq} = 0 \) and then turn back (see Figure 5). Such phenomenon is named as the “boundary reflection effect”, which may lower the loss rate predicted by QL theory. With the increasing initial equatorial pitch angle, the boundary reflection effect becomes weak due to the number decrease of electrons approaching \( \alpha_{eq} = 0 \).

[24] In the pitch angle range approximately from \( \alpha_{eq} = 10° \) to 30°, both the TP advection and diffusion coefficients show the fishbone-like oscillations. The pitch angle averaged \( \langle A_{\alpha_{eq}}^{TP} \rangle \) generally coincides with \( \langle A_{\alpha_{eq}}^{QL} \rangle \), while the pitch angle averaged \( \langle D_{\alpha_{eq}}^{TP,\alpha_{eq}} \rangle \) is sightly less than \( \langle D_{\alpha_{eq}}^{QL,\alpha_{eq}} \rangle \). The EMIC wave poses a finite quasiperiodic perturbation on the adiabatic motion of electrons before resonance, as indicated by equation (A5). Consequently, the actual resonance
locations and phases deviate from the linear ones (see Figure 6). In particular, the resonance phase determines the sign of net change in equatorial pitch angle $\Delta a_{eq}$ (see equation (A6)). Such finite perturbation effect can finally modulate $\Delta a_{eq}$ and produce the fluctuation of TP transport coefficients. For example, $\langle A_{a_{eq}}^{TP} \rangle$ is positive at $a_{eq} = 21.0^\circ/C_14$, since $17/24$ test electrons possess the resonance phases $180^\circ/C_14 < \eta < 360^\circ/C_14$; $\langle A_{a_{eq}}^{TP} \rangle$ becomes negative at $a_{eq} = 23.5^\circ/C_14$, since $13/24$ test electrons have the resonance phases $0^\circ < \eta < 180^\circ$.

[25] In the pitch angle range roughly from $a_{eq} = 30^\circ$ to $52^\circ$, the amplitudes of fishbone-like oscillations gradually increase with the increase of pitch angle. The nonlinear phase bunching process is found to occur in this pitch angle range (see the examples $a_{eq} = 39.3^\circ$, $45^\circ$ and $52^\circ$ in Figures 2 and 7). The average diffusion coefficients of TP model are smaller than those of QL model by a factor of 5 or more, and the average advection coefficients of TP model are roughly equivalent to those of QL model. Therefore, the phase bunching may lower the loss rate estimated from the QL theory. The previous theoretical estimates [Albert, 1993, 2000; Albert and Bortnik, 2009a] suggested that the phase bunching for EMIC-electron interaction produced only the positive equatorial pitch angle change. Our results here show that it can alternatively allow the equatorial pitch angle to increase (e.g., at $a_{eq} = 45^\circ$ and $52^\circ$) or decrease (e.g., at $a_{eq} = 39.3^\circ$), accounting for the oscillations of TP transport coefficients. The sign of $\Delta a_{eq}$ is determined by the bunching phase, as illustrated by equation (A6).

[26] The oscillations of TP transport coefficients in the pitch angle range from $a_{eq} = 10^\circ$ to $52^\circ$ are quasiperiodic. We believe that such quasiperiodic phenomenon is caused by the modulation of monochromatic EMIC wave before resonance (see the Appendix). The periodic variation of $\eta$ before resonance can produce the fluctuations of $a_{eq}$, the deviations of actual resonance phases and locations, and consequently the quasiperiodic oscillations of TP transport coefficients. In order to test this idea, we solve equations (4)–(7) with $B_w = 0$ (linear limit) and obtain the phase variation $\eta_{var}$ (independent of initial phase $\eta_0$) from the equator to the linear resonance point. The dependence of $\eta_{var}$ on the initial equatorial pitch angle is plotted in Figure 8. Clearly, the oscillation period of TP transport coefficients agrees well with that of $\eta_{var}$, indicating that such quasiperiodic

Figure 5. Same as Figure 2 except for the initial equatorial pitch angles $a_{eq} = 2^\circ$, $4^\circ$ and $6^\circ$. For clarity, the wave-particle phase $\eta$ is restricted to the range from $120^\circ$ to $480^\circ$. The red circles in the first row denote the boundary reflection points.
oscillations of TP transport coefficients are the realistic physical results rather than some numerical artifacts.

In the equatorial pitch angle range approximately from $\alpha_{eq} = 53^\circ$ to $62^\circ$, the signs of $\langle A_{a eq}^{TP} \rangle$ and $\langle A_{a eq}^{QL} \rangle$ are opposite, and the values of $D_{a eq}^{TP}$ and $\langle D_{a eq}^{QL} \rangle$ are generally comparable except near the singular point. Both the advection and diffusion coefficients of TP model show relatively smooth variations with pitch angle instead of those fishbone-like structures. The nonlinear phase trapping is found to occur in this pitch angle range (see the examples $\alpha_{eq} = 53.5^\circ$ and $55^\circ$ in Figures 2 and 7). Different from the phase bunching, the phase trapping only produces negative pitch angle changes, which possibly enhances the loss rate predicted by QL theory.

Secondly, we compare the transport coefficients at $E_k = 5.0$ MeV with those at $E_k = 2.0$ MeV to study the energy dependence of the differences between TP and QL models. The four phenomenons, i.e., boundary reflection effect, finite perturbation effect, phase bunching and phase trapping, can also be clearly identified at $E_k = 5.0$ MeV. Good agreements between TP and QL transport coefficients are found over a broader pitch angle range than those at $E_k = 2.0$ MeV. This trend is consistent with the predication of $R$, which is a monotonic function of $E_k$ (see Figure 1).

### 3.2. Dependence on Wave Amplitude

In this subsection, we parametrically study the dependence of the difference between TP and QL transport coefficients on the EMIC wave amplitudes.

Figure 9 shows the comparison between the transport coefficients of TP and QL models for $B_w = 0.1$ nT EMIC wave. Figure 10 plots the corresponding sample trajectories of electrons with the same initial equatorial pitch angles as those in Figure 7. It is found that the EMIC wave is too gentle to produce the phase bunching and phase trapping processes. The bounce-averaged diffusion coefficient is fairly flat throughout the resonant range, though with some small amplitude oscillations caused by the finite perturbation effect. Except near the singular points, the diffusion coefficients of TP model agree well with those of QL model. Good agreements between the TP and QL advection coefficients are observed in the low pitch angle range (where $R$ is sufficiently large). With the increase of pitch angle, the finite perturbation effect is enhanced, and consequently the...
symmetrical oscillations of TP advection coefficients around the QL ones are found to be largely strengthened. Here, the boundary reflection effect (confined within \( \sim 1^\circ \) around \( \alpha_{eq} = 0^\circ \)) appears to be unimportant. These results clearly demonstrate that the quasi-linear theory is able to well describe the wave-particle interactions when the wave amplitude is sufficiently small.

Figure 11 shows the comparison between the transport coefficients of TP and QL models for \( B_w = 10.0 \) nT EMIC wave. Figure 12 plots the corresponding sample trajectories of electrons with the same initial equatorial pitch angles as those in Figure 7. Obviously, more significant differences between the TP and QL transport coefficients occur than those for \( B_w = 2.0 \) nT. For example, the diffusion coefficients of TP model are less than those of QL model by about 1–2 magnitude throughout the pitch angle range at \( E_k = 2 \) MeV. The dominant physical processes here are still the same as those for \( B_w = 2.0 \) nT, but there are obvious migration and expansion of the equatorial pitch angle ranges dominated by the boundary reflection effect, phase bunching and phase trapping. For example, at \( \alpha_{eq} = 52.0^\circ \), the dominant physical process is the phase bunching when \( B_w = 2.0 \) nT (see Figure 7), while it becomes the phase trapping when \( B_w = 10.0 \) nT (see Figure 12). Moreover, the number percentage of electrons experiencing phase trapping and the corresponding trapping time tend to increase with the wave amplitude increasing. For example, at \( \alpha_{eq} = 53.5^\circ \), nearly one half of the test electrons are trapped experiencing 2–3 resonances for \( B_w = 2.0 \) nT (see Figure 7), while about three quarters of the test electrons are trapped experiencing \( >6 \) resonances for \( B_w = 10.0 \) nT (see Figure 12). In addition, the current strong negative advection driven by phase trapping can cause the boundary reflection effect at the pitch angles far away from the loss cone (see the examples \( \alpha_{eq} = 52.0^\circ \) and 53.5\( ^\circ \) shown in Figure 12). These results indicate that the quasi-linear theory hardly describes the radiation belt electron dynamics driven by such a large amplitude EMIC wave.

Figure 13 presents the obtained TP transport coefficients with wave amplitudes \( B_w = 0.1, 0.5, 1.0, 2.0, 5.0 \) and 10.0 nT. The increase of wave amplitude is able to enhance the boundary reflection effect and the finite perturbation effect. For the EMIC wave with sufficiently large amplitudes, the phase bunching and phase trapping characteristics can be clearly identified. According to the quasi-linear theory, both the bounce-averaged advection and diffusion coefficients are
directly proportional to the square of wave amplitudes. However, concomitant with the occurrence of boundary reflection effect, phase bunching and phase trapping, the $B_w$-dependence of $\langle A_{eq}^{TP} \rangle$ deviates from the prediction of QL theory, and $\langle D_{eq}^{TP} \rangle$ even does not increase with the increase of wave amplitude in some pitch angle ranges.

[33] Figure 14 plots the dependence of TP and QL transport coefficients on the wave amplitude at different equatorial pitch angles. The linear growth of TP transport coefficients with $B_w^2$ can be found when $B_w$ is sufficiently small. When $B_w$ exceeds some threshold values, the TP advection coefficients start to deviate from the linear growth with $B_w^2$, meanwhile the TP diffusion coefficients become saturated. The black lines and symbols represent the transport coefficients at the loss cone boundary. The TP diffusion coefficients reach the saturation level when $B_w$ approximately equals 1 nT at $E_k = 2$ MeV and 2 nT at $E_k = 5$ MeV, resulted from the boundary reflection effect. The blue lines and symbols represent the mean transport coefficients over the pitch angle range from $\alpha_{eq} = 15^\circ$ to $40^\circ$ at $E_k = 2$ MeV, and from $\alpha_{eq} = 15^\circ$ to $50^\circ$ at $E_k = 5$ MeV. The mean TP diffusion coefficients saturate at about $B_w = 3$ nT for both $E_k = 2$ and 5 MeV due to the wide presence of phase bunching. The red symbols represent the maximum values of $\langle A_{eq}^{TP} \rangle$ and $\langle D_{eq}^{TP} \rangle$. These maximum locations are generally around the equatorial resonance points (where $R = 0$). The negative advection is related to the finite perturbation effect when $B_w < 0.5$ nT (see Figure 9), and associated with the phase trapping when $B_w > 0.5$ nT (see Figure 11). Across the boundary $B_w = 0.5$ nT, the variation rates of $\langle A_{eq}^{TP} \rangle$ for both $E_k = 2$ and 5 MeV electrons obviously decrease. The corresponding threshold amplitude for the saturation of TP diffusion coefficients is about $B_w = \ldots$

**Figure 8.** Dependence of bounce-averaged test-particle (black) and quasi-linear (red) pitch angle (top) advection and (bottom) diffusion coefficients on the initial equatorial pitch angle $\alpha_{eq}$; dependence of phase variation $\eta_{var} = \eta_r - \eta_0$ (blue) on the initial equatorial pitch angle $\alpha_{eq}$, where $\eta_0$ and $\eta_r$ are the phases at the equator and linear resonance point.

**Figure 9.** Same as Figure 4 except for $B_w = 0.1$ nT EMIC wave.
5 nT at $E_k = 2$ MeV and $B_w = 10$ nT at $E_k = 5$ MeV. Such saturation may be attributed to the boundary reflection effect (see the examples $\alpha_{eq} = 52.0^\circ$ and $53.5^\circ$ in Figure 12).

4. Conclusions and Discussions

[34] The EMIC wave is widely believed to play an important role in the loss of relativistic outer radiation belt electrons [Thorne and Kennel, 1971; Horne and Thorne, 1998; Summers et al., 1998; Bortnik et al., 2006; Su et al., 2011a]. Such loss process has been investigated by numerous quasi-linear simulations [e.g., Li et al., 2007; Jordanova et al., 2008; Shprits et al., 2009a, 2009b; Su and Zheng, 2009; Su et al., 2010a, 2011b, 2011c]. Recent gyro-averaged test-particle simulations [Albert and Bortnik, 2009a] have preliminarily revealed the nonlinear characters of the interactions between large amplitude EMIC waves and relativistic electrons. In this study, the bounce-averaged pitch angle advection and diffusion coefficients for parallel-propagating monochromatic EMIC waves are evaluated based on the test-particle simulations, and then the comparison of transport coefficients between TP and QL models is made to investigate the influence of nonlinear processes. Our principal conclusions are as follows:

[35] 1. Apart from the frequently invoked phase bunching and phase trapping, two other nonlinear processes, named boundary reflection effect and finite perturbation effect, are introduced for the EMIC-electron interaction. The former refers to the “reflection” of test electrons at the equatorial pitch angle boundary $\alpha_{eq} = 0^\circ$. The latter refers to the deviation of actual resonance locations and phases from the linear ones due to the finite perturbation of waves. The dimensionless parameter $R$ (characterizing the competition between wave-induced and adiabatic motions) is able to roughly determine the occurrence regions of finite perturbation effect, phase bunching and phase trapping [Albert and Bortnik, 2009a]. For the current EMIC-electron interaction, the $R$ values are found to generally satisfy the relation: $R_1 \sim 1 \sim R_2 \sim R_3$, where the subscripts 1, 2, and 3 represent the finite perturbation effect, phase bunching and phase trapping, respectively. However, the presence of boundary reflection effect can not be predicted simply, since it can occur in the region with large (near the loss cone) or small (related to the strong phase trapping) $R$ values.

[36] 2. For a EMIC wave with typical amplitude $B_w = 2$ nT, the boundary reflection effect, finite perturbation effect, phase bunching and phase trapping processes are found to occur sequentially from small to large equatorial

![Figure 10. Same as Figure 7 except for $B_w = 0.1$ nT EMIC wave.](image-url)
pitch angles. The results from TP model agree well with those from QL model over a broader pitch angle range at higher kinetic energies. The TP transport coefficients related to the finite perturbation effect and phase bunching show fishbone-like oscillations, while those associated with the boundary reflection effect and phase trapping behave smoothly. The pitch angle averaged finite perturbation effect yields slight differences between the transport coefficients of TP and QL models. The boundary reflection effect and phase bunching tend to lower the loss rate predicted by QL theory, producing an average reduction of >80% in the diffusion coefficients but a small change in the corresponding average advection coefficients. In contrast, the phase trapping tends to increase the loss rate predicted by QL theory, causing continuous negative advection toward the loss cone and a minor change in the corresponding diffusion coefficients.

For an EMIC wave with sufficiently small amplitude $B_{w} = 0.1 \text{ nT}$, the transport coefficients of TP model are found to agree well with those of QL model almost over all the pitch angles, demonstrating the reliability of QL theory under such circumstance. According to the QL theory, the transport coefficients grow linearly with the square of wave amplitude $B_{w}^2$. As the amplitude increases, the boundary reflection effect, phase bunching and phase trapping begin to occur. Consequently, the TP advection coefficients deviate from the linear growth with $B_{w}^2$, and the TP diffusion coefficients become saturated with the amplitude approaching 1 nT or above. For the EMIC wave with very large amplitude $B_{w} = 10.0 \text{ nT}$, the diffusion coefficients of TP model are less than those of QL model by 1–2 orders of magnitude over all the pitch angles, and the continuous negative advection induced by the phase trapping can occur over a broad pitch angle range of ~20°. The current results suggest that these nonlinear processes can significantly affect the transport coefficients, which need to be taken into account in the future simulations of radiation belt dynamics driven by the EMIC waves.

The EMIC waves are often observed as broadband spectrums [e.g., Anderson et al., 1992a, 1992b] or narrowband packets with rising tones [Pickett et al., 2010]. These waves are able to occur over a broad spatial region [Meredith et al., 2003], where the background plasma properties show significant radial and latitudinal variations during storms [e.g., Sheeley et al., 2001; Tsyganenko and Sitnov, 2005; Denton et al., 2006]. Previous works have pointed out the importance of field-aligned density variation [Su et al., 2009a] and non-dipole field [e.g., Ni et al. 2011b] on the evaluation of QL diffusion coefficients. These factors may also quantitatively affect the nonlinear wave-particle interaction processes. Here we primarily analyze the case for monochromatic EMIC wave in a dipole magnetic field with constant field-aligned density, and more realistic simulations on the cyclotron resonance between radiation belt electrons and EMIC waves are left for future studies.

The minimum resonant energy of EMIC sensitively depends on the background density and wave-frequency [Summers and Thorne, 2003]. To resonate with <MeV electrons, the EMIC wave must have frequency close to the ion gyro-frequency and occur in the high density region. In contrast, the plasmaspheric hiss and chorus waves can resonate with electrons over a wide energy range [e.g., Ni et al., 2008, 2011a, 2011b; Shprits et al., 2009b; Su et al., 2009b, 2010c; Xiao et al., 2009], whose contributions to the loss of radiation belt electrons have been investigated through the QL...
simulations [e.g., Thorne et al., 2005; Li et al., 2007; Su et al., 2010a, 2011a, 2011c]. The recent TP simulations [Bortnik et al., 2008] show that the large amplitude chorus waves can cause both the loss (via phase bunching) and energization (via phase trapping) of radiation belt electrons. The competition and cooperation of these waves on the radiation belt electron dynamics should be explored in the future work.

Appendix A: Pitch Angle Change Rate

[40] From (4) and (5), we have the local pitch angle change rate [Inan, 1987]

$$\frac{d\alpha}{dt} = \cos \alpha \frac{d\tan \alpha}{dt} = \cos \alpha \frac{d^2 \tan \alpha}{dt^2} \left( \frac{p_\perp}{p} \right)$$

$$= \frac{1}{p^2} \left( \frac{dp_\perp}{dt} p_\parallel - \frac{dp_\parallel}{dt} p_\perp \right)$$

$$= \frac{eB_w}{p^2} \left( \frac{\omega}{k} \frac{p_\parallel}{\gamma m_e} \right) \frac{p_\perp}{p} - \frac{p_\parallel}{2 \gamma m_B c_s^2} \sin \eta + \frac{p_\parallel}{\gamma m_e} \frac{\partial B}{\partial s}. \quad (A1)$$

On the righthand side, the two terms represent the wave-induced and adiabatic change rates. The relation between local and equatorial pitch angles can be written as

$$\sin^2 \alpha = \sin^2 \alpha_{eq} \frac{B}{B_{eq}}. \quad (A2)$$

Differentiating (A2) gives

$$\frac{d\alpha}{dt} = \tan \alpha \frac{d\alpha_{eq}}{dt} + \frac{p_\parallel}{2 \gamma m_B c_s^2} \frac{\partial B}{\partial s}. \quad (A3)$$

where use has been made of

$$\frac{dB}{dt} = \frac{\partial B}{\partial t} + \frac{p_\parallel}{\gamma m_e} \frac{\partial B}{\partial s}.$$ \quad (A4)

Using (A1) and (A3), we obtain the change rate of equatorial pitch angle

$$\frac{d\alpha_{eq}}{dt} = \frac{eB_w}{p^2} \tan \alpha_{eq} \left( \frac{\omega}{k} \frac{p_\parallel}{\gamma m_e} \right) \frac{p_\perp}{p} - \frac{p_\parallel}{2 \gamma m_B c_s^2} \sin \eta. \quad (A5)$$

Figure 12. Same as Figure 7 except for $B_w = 10$ nT EMIC wave. For clarity, only the phase trapping trajectories are plotted in Figures 12f and 12i. The red circles in the first row denote the boundary reflection points.
Figure 13. Profiles of bounce-averaged test-particle pitch angle (top) advection and (bottom) diffusion coefficients at kinetic energies (left) $E_k = 2$ and (right) 5 MeV for EMIC waves with indicated amplitudes.

Figure 14. Wave-amplitude dependence of bounce-averaged test-particle (circles) and quasi-linear (lines) pitch angle (top) advection and (bottom) diffusion coefficients for (left) $E_k = 2$ and (right) 5 MeV electrons. The black lines and circles represent the transport coefficients at the loss cone boundary. The blue lines and circles represent the mean transport coefficients over the pitch angle range from $\alpha_{eq} = 15^\circ$ to $40^\circ$ at $E_k = 2$ MeV, and from $\alpha_{eq} = 15^\circ$ to $50^\circ$ at $E_k = 5$ MeV. The red circles represent the maximum values of $-\left\langle A_{ai\alpha}^{TP} \right\rangle$ and $\left\langle D_{ai\alpha\alpha}^{TP} \right\rangle$. 
Considering (6), $\eta$ is a quasiperiodic function before resonance. The monochromatic wave can cause the quasiperiodic fluctuation of $\alpha_{eq}$, and consequently the actual resonance locations and phases deviate from the linear ones. Substituting (8) into (A5) yields the change rate of equatorial pitch angle at the resonance point

$$\left(\frac{d\alpha_{eq}}{dt}\right)_R = -\frac{E_p}{g^2} \tan \alpha_{eq} \left(\frac{\Delta \rho_p}{k^2} \tan \alpha + \frac{\Delta p}{m_i} \sin \eta\right).$$

(A6)

For the current EMIC-electron interaction, $p_i/k$ is positive. Hence, the sign of net change in equatorial pitch angle $\Delta \alpha_{eq}$ is determined by the resonance phase $\eta$. If $180^\circ < \eta < 180^\circ$ at the resonance point, $\Delta \alpha_{eq}$ is negative; if $180^\circ < \eta < 360^\circ$ at the resonance point, $\Delta \alpha_{eq}$ is positive.

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