Effect of current sheets on the power spectrum of the solar wind magnetic field using a cell model

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Abstract

A puzzling observation of solar wind MHD turbulence is the often seen Kolmogorov scaling of $k^{-3/2}$, even though the solar wind MHD turbulence is dominated by Alfvénic fluctuations. Recently Li et al. (2011) proposed that the presence of current sheets may be the cause of the Kolmogorov scaling. Here, using a cell model of the solar wind we examine the effect of current sheets on the power spectrum of the solar wind magnetic field. We model the solar wind as multiple cells separated by current sheets. We prescribe the spectra of turbulent magnetic field in individual cells as IK-like and examine the spectra along trajectories that cross multiple boundaries. We find that these spectra become softer and are consistent with the Kolmogorov-scaling. Our finding supports our recent proposal of Li et al. (2011).

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1. Introduction

Solar wind provides a natural site for studying magnetohydrodynamics (MHD) turbulence. In the past years, tremendous data from various spacecraft on plasma density, flow speed and magnetic field have advanced our understanding of the MHD turbulence significantly. For extensive reviews, see (Tu and Marsch, 1995; Goldstein et al., 1995; Bruno and Carbone, 2005).

One puzzling observation concerns the power spectra of the turbulent magnetic field and flow velocity: $\delta B$ and $\delta \vec{v}$. In the first MHD turbulence theory developed by Iroshnikov (1964), Kraichnan (1965) (hereafter IK theory), the energy cascade is mediated by Alfvén wave packets, which leads to a power spectrum that scales as $k^{-3/2}$. Observations (Goldstein, 2001; Horbury et al., 2005; Leamon et al., 1998; Smith et al., 2006), however, often show power laws of $\sim k^{-5/3}$ that agree with the Kolmogorov (1941) theory (hereafter K41 theory). This is puzzling because the K41 theory was proposed to explain hydrodynamic turbulence where the energy is cascaded, in the inertial range, from large scales to small scales through eddy motions.

Oughton and Matthaeus (2005) suggested that the often seen K41-like scaling of the solar wind magnetic field and velocity spectra has to do with the obliquity of Alfvén fluctuations in the solar wind. These authors argued that when the Alfvén fluctuation is highly perpendicular, the decorrelation time due to Alfvén mode cascading $\tau_A$ will become larger than the decorrelation time due to non-linear effect $\tau_{NL}$ so the cascading is dominated by non-linear effects where a scaling of K41 will emerge. It has been pointed out (e.g. see (Shebalin et al., 1983; Chandran, 2008)) that highly perpendicular Alfvén waves are favored if the cascading occurs mainly along $k_\perp$. 

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Using the technique of conditional wavelet analysis, Veltri et al. (1999), Salem et al. (2007, 2009) showed that the magnetic field and velocity components of the solar wind can exhibit K41 and IK scaling at the same time. Chapman and Hnat (2007) examined the structure functions of the solar wind $\delta v$ and $\delta B$ and showed that the fluctuation in the solar wind velocity is a linear superposition of compressive and hydrodynamic-like turbulence that obeys the K41 scaling and Alfvénic turbulence that obeys the IK scaling. These authors further argued that the turbulent solar wind may be comprised of two weakly interacting components: one from the process that generates the solar wind at the corona, having an IK scaling, and the other intrinsically evolves in the high Reynolds number solar wind, having a K41 scaling.

Note, the IK model ignored the fact that the weak MHD turbulence is intrinsically anisotropic due to the background magnetic field $\overline{B}_0$. Since Sridhar and Goldreich (1994), the fact that a weak turbulence is anisotropic and has a power spectrum $\sim k^{-2}$ has been broadly discussed analytically (e.g. Ng and Bhattacharjee, 1996; Galtier et al., 2000, 2002; Biskamp, 2003) and confirmed numerically (see e.g. Perez and Boldyrev (2008)). Therefore, a $k^{-1.5}$ spectrum of the solar wind MHD turbulence implies that the solar wind MHD turbulence is not in the weak MHD regime. In fact, theoretical studies (Boldyrev, 2006; Boldyrev and Perez, 2009) indicated that a $-3/2$ spectrum can emerge in a strong MHD turbulence regime that is more appropriate to the solar wind MHD turbulence.

Recently, using a wavelet analysis, Horbury et al. (2008) and Podesta (2009) showed that when the local mean magnetic field direction is nearly parallel or anti-parallel with the solar wind flow, the power spectrum approaches $f^{-2}$, agreeing with the critical balance theory of Goldreich and Sridhar (1995) (Hereafter GS95). Extending the study of Horbury et al. (2008), Forman et al. (2011) performed a detailed study on the dependence of the spectral index on the angle $\theta_B$ between the local magnetic field direction and the flow direction. They found that the power spectrum in the inertial range is highly anisotropic with $f$-behaving like $f^{-5/3}$ at $\theta_B \sim 90^\circ$ and $f^{-2}$ at $\theta_B \sim 0^\circ$, using the second-order structure function, Luo and Wu (2010) also suggested that the spectral index can vary with $\theta_B$. In another study, Roberts (2010) examined how solar wind velocity spectra evolve from 0.3 to 5 AU using data from the Helios and Ulysses spacecraft. Roberts (2010) found that the spectrum of the velocity at $r < 1$ AU is in general flatter than that of the magnetic field. Beyond 1 AU when the wind is relatively non-Alfvénic, both spectra become equal (at $-5/3$). Roberts (2010) suggested that the solar wind MHD turbulence is still evolving at 1 AU and a simple “inertial range” with uniform spectral properties is unrealistic for the solar wind. However, in a recent study, Boldyrev et al. (2011) demonstrated that in-situ observations at 1AU of the solar wind MHD turbulence are in remarkable agreement with numerical simulations of fully developed, steady-state MHD turbulence. Using WIND Magnetic Field Investigation (MFI) data, Podesta and Borovskv (2010) found that the spectral slope for the total energy (kinetic and magnetic) is correlated with the normalized helicity $\sigma$, such that when $\sigma \sim 1$ an IK scaling is found and when $\sigma \sim 0$ a K41 scaling is found.

Based on an analysis of 3-year worth data from Ulysses, Li et al. (2011) suggested that current sheet (or the absence of it) in the solar wind is the cause of the K41 (or the IK) scaling of the solar wind magnetic field power spectra. By identifying ten 24-h periods that either contain no current sheets or contain the most number of current sheets, Li et al. (2011) found that depending on whether or not current sheets are present in the periods of study, either K41-scaling or IK-scaling will emerge. Li et al. (2011) further suggested that the reason that solar wind MHD observations often find a K41 scaling is because that current sheets occur very frequently in the solar wind. In another study, using more than eight-year’s data from ACE observation, Borovsky (2010) examined the effect of the strong discontinuities to the power spectrum of the solar wind. By constructing an artificial time series that preserves the timing and amplitudes of the discontinuities, Borovsky (2010) showed that the strong discontinuities can produce a power-law spectrum in the inertial sub-range with a K41-type scaling.

The current sheet in the (Li et al., 2011) is a 2D structure where the magnetic field direction changes significantly from one side to the other. Current sheets are common in the solar wind and are a major source of solar wind MHD turbulence intermittency (Salem et al., 2007; Li, 2008). However, the origin of these structures is still under debate. They may emerge from nonlinear interactions (Zhou et al., 2004; Chang et al., 2004) or represent relic “magnetic walls” that originate from the surface of the Sun (Bruno et al., 2001; Borovsky, 2006; Li, 2008). In the first picture, structures can emerge locally from MHD turbulence. Recently Greco et al. (2009a,b) compared Advanced Composition Explorer (ACE) solar wind data and numerical simulations of magnetohydrodynamic (MHD) turbulence to show that there is a good agreement in the waiting-time analysis and probability distribution functions between the observation and the simulation. Greco et al. (2009a) argued that these similarities suggest that current sheets can quickly emerge in MHD turbulence. In the latter picture, plasmas in the solar wind are bundled in “spaghetti-like” flux tubes. Li (2007, 2008) developed a method that is based on the $\zeta$-scaling properties of the angle $\beta = \cos^{-1}(\overline{B}(t) \cdot \overline{B}(t+\zeta))$ to identify current sheets in the solar wind. Using this method Li et al. (2008) studied the magnetic field data from the Cluster spacecraft for two periods that are in the solar wind and in the Earth’s magnetotail respectively. They found that while there is clear signature of current sheets in the solar wind, similar structures could not be identified in the Earth’s magnetosphere. The study of Li et al. (2008) therefore is consistent with the “flux tube” picture of the solar wind as firstly proposed by Bruno et al. (2001). Extending Li (2007, 2008), Miao et al.
Miao et al. (2011) developed an automatic current sheet identification routine. Using this routine, Miao et al. (2011) analyzed more than 3 years magnetic field data from Ulysses spacecraft and identified more than 28000 current sheets. The study of (Miao et al., 2011) showed that current sheets are common in the solar wind.

The existence of current sheets can affect the transport of solar energetic cosmic rays. The first such study has been done by Qin et al. (2008), who used a toy “cell model” of the solar wind in modeling the current sheet structures and showed that the presence of these current sheets can affect the transport of energetic cosmic rays along the direction perpendicular to the background magnetic field. The toy “cell model” constructed in Qin et al. (2008) did not attempt to represent the realistic solar wind that is observed at 1 AU. An improved cell model of the solar wind was introduced in Li and Qin (2011). In this work we follow the (Li and Qin, 2011) and construct a realistic solar wind cell model to study the effect of current sheets on the power spectrum of the solar wind magnetic field.

2. Model description

The fundamental assumption of the cell model is that the solar wind is composed of small cells of random size, with adjacent cells being separated by current sheets. We ignore the thickness of the current sheets and model the change of magnetic field directions as sharp kinks (Qin et al., 2008). In every individual cell, the magnetic field consists of a uniform mean magnetic field \( B_0 \) and a turbulent magnetic field \( \delta B \). The direction of \( B_0 \) is assumed to be different from the underlying large scale background field \( B_0 \) direction; and the magnitude of \( \delta B \) is assumed to be the same magnitude as \( B_0 \). The individual cells are obtained using a recursive procedure (Li and Qin, 2011): we assume the solar wind in the model occupies a space \( M_0 \) and we first cut \( M_0 \) into two small pieces; we then recursively apply the cutting procedure to a level of \( k \) where a total number of \( N = 2^k \) convex polyhedron cells are obtained. The schematics of the cutting procedure in 2D is shown in Fig. 1. Here we assume \( k = 4 \). The upper left panel corresponds to the first step, where a thick black line divides the current sheet free solar wind (i.e. \( M_0 \)) to two smaller regions. In the second step, corresponding to the upper right panel, two more current sheets, represented by the solid black line further divides \( M_0 \) to four pieces. Similarly, the lower left panel corresponds to the third step where current sheets are represented by dashed line, and the lower right panel corresponds to the fourth (and the last) step where current sheets are represented by double-dotted lines. Once individual cells are obtained, we decide the local background magnetic field \( B_0^L \) from the large scale background magnetic field \( B_0 \), respectively. In Fig. 1, the arrows in all the small regions in the last panel represent the “local” magnetic field direction. The large scale background magnetic field is shown as \( B_0 \).

To best describe a real solar wind at 1 AU, we follow (Li and Qin, 2011) and use the following formula for the probability density of \( \beta \) and \( \phi \),

\[
p(\beta) = \frac{1 - \eta}{\sqrt{\pi x_1}} \exp \left[ -\left( \frac{\beta}{x_1} \right)^2 \right] + \frac{\eta}{\sqrt{\pi x_2}} \exp \left[ -\left( \frac{\beta}{x_2} \right)^2 \right],
\]

\[
\eta(\phi) = 1/360
\]

where \( \eta = 0.036 \), \( x_1 = 16.9^\circ \), and \( x_2 = 45.5^\circ \). Eq. (1) describes the “bending” of the local magnetic field direction from the large scale background magnetic field direction and is given by two Gaussian distribution. Eq. (2) describes the “twisting” of the local magnetic field direction from the large scale background magnetic field and is given by a uniform random distribution. We also normalize our cell model to \( L \times L \times L \) where \( L = 26.5 \) AU. To describe the real solar wind at 1 AU, we fit the modeled distribution of the cell size to the waiting time distribution between current sheets as well as the modelled distribution of the deflection angle of magnetic field direction across a current sheet to that obtained from observation. For the above choice of \( L \), the fitting to 1 AU observation by ACE (Miao et al., 2011; Li and Qin, 2011) suggests that a \( k = 13 \) is needed.
3. Results

With this solar wind model, we examine the effect of the current sheets on the power spectrum of solar wind magnetic field by flying an imaginary spacecraft and calculate the power spectra along trajectories that have various angles \( \theta \) between the trajectories and the background magnetic field \( B_0 \). At 1 AU, the radial direction and the background magnetic field direction has an angle of \( \sim 45^\circ \). In each individual cells, we prescribe a slab turbulence with a spectral amplitude given by (for details see Qin et al. (2002a,b)),

\[
C_{slab}^{xx} = \frac{C_{slab}}{k_z^2}.
\]

(3)

In this work, we assume that the input turbulence in individual cells are IK-like, therefore we set \( v = -1.5 \). The magnetic field in the real space is obtained from the spectrum using a discrete Fast Fourier Transform method,

\[
b^{slab} = \sum_{k_z} b(k_z) \exp ik_z z
\]

(4)

We chose \( C_{slab} \) such that the amplitude of \( \delta B = 0.5B_0 \). This is in agreement with typical 1 AU observations of \( B_0 \sim 4.12 \) nT and \( \delta B_{slab}^2 \sim 4.0 \) nT\(^2\).

The proposal of Li et al. (2011) suggested that even if the spectrum in individual cells are IK-like, as long as the trajectory of a spacecraft crosses multiple current sheets, the power spectrum of the magnetic field along that trajectory will become Kolmogorov-like.

Fig. 2 plots the spectra along four trajectories with \( \theta = 0^\circ \), \( \theta = 15^\circ \), \( \theta = 30^\circ \) and \( \theta = 45^\circ \). The upper left panel is for \( \theta = 0^\circ \); the upper right panel is for \( \theta = 15^\circ \); the lower left panel is for \( \theta = 30^\circ \) and the lower right panel is for \( \theta = 45^\circ \). The x-axis is frequency in Hz and the y-axis is the power in \( nT^2/Hz \). To guide the eyes, we show \( \sim k^{-5/3} \) (the K41-like) lines in the figure. The fitted spectral indices (for the range of 0.2–20 Hz) are also included in the figure.

It is clear from Fig. 2 that in all four cases, the power spectra are Kolmogorov-like, even though the power spectrum in individual cells are IK-like. In obtaining the power spectra, we used the common Welch method by averaging periodograms (e.g., Porat, 1997) that uses the technique of involving segmentation of time-series data.

Fig. 3 plots the power law index \( \gamma \) as a function of \( \theta \). For each \( \theta, \gamma \) is obtained by ensemble averaging a total of 20 trajectories that have different \( \phi \)s given by,

\[
\phi = \frac{i \pi}{10}
\]

with \( i = 0 \) to 19 and the variances as the uncertainties. Clearly, the spectral indices are consistent with 5/3 for all \( \theta \)s.

Figs. 2 and 3 suggest that the existence of current sheet in the solar wind can significantly affect the power law spectrum of the solar wind magnetic field. A turbulence that is locally IK-like can behave as Kolmogorov-like when the trajectories from which the turbulence power spectrum is obtained include many current sheets. This is in agreement with the recent finding of Li et al. (2011).
4. Discussion and conclusion

Recent studies of solar wind MHD turbulence suggested that current sheets are common structures in the solar wind. Magnetic field direction undergoes abrupt changes at these structures. We recently proposed that the existence of these structures may explain why a Kolmogorov type of scaling is common in the solar wind (Li et al., 2011).

In this paper, we used a cell model of the solar wind to examine our earlier proposal. We found that an initially IK-like turbulence having a slab geometry will evolve into a K41-like spectrum when current sheets are added to the system. For the given parameters that resemble the solar wind at 1 AU, we found that the average power law indices for trajectories having \( \theta = 0^\circ, 15^\circ, 45^\circ \) and \( 75^\circ \) degrees from the \( B_0 \) direction are \(-1.72, -1.67, -1.66 \) and \(-1.68 \), respectively. All of these spectra are consistent with the Kolmogorov-scaling. Our results support our recent proposal of Li et al. (2011) and suggest that the existence of current sheets in the solar wind can alter the solar wind MHD turbulence power spectrum. We do not consider the origin of the current sheet in this paper except to note that they can either arise from non-linear interactions of the solar wind MHD turbulence or are relic structures that originated from the surface of the Sun. K41 scaling of the solar wind MHD turbulence may emerge in either case. If the current sheets result from the non-linear interaction of the solar wind MHD turbulence, then we expect the cascading is dominated by non-linear interactions in current sheet abundant intervals and Alfvénic modes in current sheet free intervals. If the current sheets are relic structures rooted from the surface of the Sun, then being the boundaries of flux tubes originated from structures on the surface of the Sun, current sheets may be regarded as the results of relative fluid motion between large scale structures, (e.g. supragranules), therefore, at the scale of the flux tubes, resulting a fluid-like (i.e. K41) turbulence. In this picture, a period that is abundant with current sheets corresponds to a period where the scale of the flux tubes are smaller. Strong discontinuities may lead to a K41 scale has also been discussed by Borovsky (2010) who, by constructing an artificial time series data containing strong discontinuities, demonstrated that a power-law spectrum consistent with a K41-type scaling can emerge. We point out that identification of current sheets is crucial for analyses that require the identification of local magnetic field directions such as those in Horbury et al. (2008), Podesta (2009), Luo and Wu (2010), Forman et al. (2011). This is because the magnetic field directions change abruptly cross current sheets. Therefore one has to be careful in obtaining and interpreting the local magnetic field direction if there are current sheets within the period of consideration.

The transport of solar energetic particles in the inner heliosphere is modulated by the solar wind MHD turbulence power spectrum through particle wave interaction. Therefore, if the existence of current sheets in the solar wind can affect the solar wind MHD turbulence power spectrum, it will also affect the transport of solar energetic particles.

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