Invited Paper

Single-pixel complementary compressive sampling spectrometer

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Abstract

A new type of compressive spectroscopy technique employing a complementary sampling strategy is reported. In a single sequence of spectral compressive sampling, positive and negative measurements are performed, in which sensing matrices with a complementary relationship are used. The restricted isometry property condition necessary for accurate recovery of compressive sampling theory is satisfied mathematically. Compared with the conventional single-pixel spectroscopy technique, the complementary compressive sampling strategy can achieve spectral recovery of considerably higher quality within a shorter sampling time. We also investigate the influence of the sampling ratio and integration time on the recovery quality.

Keywords:
Single-pixel spectrometer
Compressive sampling
Complementary measurement
Spectral recovery

1. Introduction

The spectrometer is one of the most versatile instruments available for the analysis of the attributes of matter, and this device is widely used in a variety of research fields such as chemistry, biology, and astronomy. The majority of compact spectrometers are based on an optics dispersive component (prism or grating), in which the spectrum is generally measured by a linear detector array. A charge-coupled device (CCD) array is extremely complex, bulky, and expensive. Array can be reduced to that of a single unit, which enables the use of exotic detectors.

Many related CS applications have emerged in the fields of spectrometry and spectral imaging. In these applications, in order to realize spectral compressive sampling using linear optical elements, non-negative sensing matrices are adopted. However, these matrices do not satisfy the restricted isometry property (RIP) condition that is essential to ensuring CS robustness; this is not favorable to high-quality spectral recovery. This problem has been termed “the non-negativity problem” by some scholars, and Roummel et al. have proposed a method known as “mean subtraction” to overcome this issue. However, in the proposed method, the mean is estimated with relatively poor accuracy. Therefore, the mean subtraction method does not essentially improve the CS reconstruction quality.

Using the previously proposed single-pixel camera structure as a basis, we design a new single-pixel spectrometer. In our design, we use the complementary compressive sampling strategy to solve the non-negativity problem. With this optical structure, the complementary compressive sampling strategy achieves significantly higher-quality spectral recovery than other methods within a shorter sampling time. This paper is structured as follows. In Sections 2 and 3, the CS theory and its application to signal reconstruction are respectively explained. The proposed experimental setup is described in Section 4, and the results of evaluation experiments are presented in Section 5. In Section 6, certain aspects of the findings are discussed, while the conclusions are
given in Section 7.

2. Compressive sampling theory

CS does not measure signal elements directly, but instead measures the inner products of the signal and a set of test functions named sensing matrices. We take \( x \) (a length-n column vector) as an original signal and \( A \) as an \( m \times n \) \((m < n)\) matrix. Then, \( A \) can project \( x \) on an \( m \)-dimensional set of observations \( y \), such that
\[
y = Ax + e,
\]
where \( e \) is noise. As \( m < n \), this is generally an ill-posed problem with an infinite number of possible candidate solutions. Nevertheless, CS theory provides a set of conditions restricting \( A, x \) and \( e \). If these conditions are satisfied, accurate reconstruction can be achieved by solving a simple convex optimization problem \cite{4999917}. First, \( x \) is required to be sparse or compressible under a certain basis. Second, \( A \) must satisfy the RIP. For example, if the entries of \( A \) are independent and identically distributed according to
\[
A_{ij} = \begin{cases} 
    m^{-1/2} & \text{with probability } 1/2 \\
    -m^{-1/2} & \text{with probability } 1/2
\end{cases}
\]
(2)
then \( A \) has a high possibility of satisfying RIP \cite{10.1137/080731359,10.1137/080731344,10.1137/080731367}. Third, \( e \) must be zero or bounded, so the spectrometer must be designed such that the minimum possible \( e \) is obtained (note that increasing the integer time is an effective method of reducing \( e \)).

3. Improving reconstruction performance using complementary measurement

Pseudorandom sensing matrices such as those described in Section 2, which satisfy the RIP, provide theoretical guarantees regarding the reconstruction accuracy in the presence of Gaussian or bounded noise. However, approximately half of the elements in these matrices are negative, and it is impossible to construct such a system using linear optical elements. The digital micro-mirror device (DMD) is one such optical element, which is often used to modulate light \cite{10.1109/JSTQE.2010.2076750,10.1109/ICIP.2008.4711311}. The DMD consists of a number of micro-mirrors, and each micro-mirror rotates about a hinge and can be independently actuated to two positions oriented at \( +12^\circ \) (according to 1 in binary matrix) or \(-12^\circ\) (according to 0); thus, light falling on the DMD may be reflected in two directions depending on the mirror orientation. Matrices fed into the DMD must be non-negative, for example, pseudorandom 0/1 binary matrices with 0.5 mean. Although non-negative matrices are physically realizable, such observations cannot be used directly to realize accurate CS recovery.

In this paper, we adopt a complementary measurement method to address the non-negativity problem. In this method, two complementary measurements are performed in a single CS reconstruction. We suppose that \( A_q \) and \( A_e \) are two pseudorandom 0/1 binary matrices with 0.5 mean, and their elements satisfy the relationship
\[
A_q(i,j) = 1 - A_e(i,j),
\]
i.e., \( A_q \) and \( A_e \) are complementary matrices. Then, we obtain two measurement results \( y_q \) and \( y_e \) for \( A_q \) and \( A_e \) respectively, where
\[
y_q = A_q x + e_q,
\]
\[
y_e = A_e x + e_e,
\]
by subtracting Eq. (5) from Eq. (4), we obtain the complementary differential measurement result \( y_q - y_e \).
\[
y_q - y_e = (A_q - A_e)x + (e_q - e_e) = A_q x + e_q - A_e x - e_e.
\]
Here, \( A_q \) is a pseudorandom \(-1/1\) binary matrix with zero mean, i.e., \( A_q \) completely satisfies the RIP condition and has the same distribution as \( A_e \), except \(-1\) takes the place of 0. Thus, by solving Eq. (6) using an appropriate CS algorithm, we can, in theory, realize accurate signal reconstruction. It is possible to reconstruct \( x \) by solving the optimization problem
\[
\min_x \| y_q - A_q x \|_p^p + \tau \| x \|_0
\]
(7)
where \( \tau \) is a constant scalar and \( \| \cdot \|_0 \) represents the \( l_0 \) norm, defined as \( (\| x \|_0)^p = \sum_{i=1}^n \| x_i \|_p^p \).

4. Experimental setup

A schematic diagram of the experimental setup is given in Fig. 1. Here, Optical source is focused and passes through a slit entrance, which is then collimated by the L1 lens. The collimated light is dispersed by a fixed diffraction gating along different angles in space according to its wavelength. Under the influence of the L2 lens, the spectrum is imaged onto the DMD. We employ a Texas Instruments (TI) DMD which consists of 1024 \( \times \) 768 micro-mirrors, each of which is 13.68 \( \times \) 13.68 \( \mu \)m in size. As the spectral image is a one-dimensional signal, we treat each DMD column as one unit. This means that the spectral signal \( x \) is separated into 1024 spectrum bands \((n=1024)\). When the DMD columns are rotated according to the pseudorandom 0/1 sensing matrix that is fed into the device, randomly selected parts of \( x \) are reflected by \( +12^\circ \) and collected by L3 lens. The total intensity of the selected light is measured by a single PMT detector after it passes through L3.

In Fig. 1, the slit width is 100 \( \mu \)m, and the focal lengths of L0, L1, L2, and L3 are 30, 30, 100, and 50.8 mm, respectively. The gating is produced by the Thorlabs Company, and has a 500 nm blazing wavelength, with 1800 lines per mm. The angle of incidence of the light on the grating is 60\(^\circ\). Theoretically, the average optical spectral resolution is approximately 0.92 nm. The PMT is of photon-counting type, and is produced by the Hamamatsu Company.

Note that we employ a one-arm-detector structure in our experiment, i.e., only the light deflected by the \(+12^\circ\) direction of the DMD is measured by the PMT. Thus the light deflected by \(-12^\circ\) is discarded. If we perform a positive measurement, the light deflected by \(-12^\circ\) corresponds to a negative measurement.
Therefore, we can employ a double-arm-detector structure, which adds the same optic components to the light deflected by $-12^\circ$ as is added to the light sent in the $+12^\circ$ direction. Therefore, in one DMD modulation time sequence, we can complete both positive and negative measurements. Thus, the total sampling time is halved.

5. Experimental results

In the experiment conducted in this study, the optical source corresponded to monochromatic light of 632.8 nm, the half-height width of which was 10 nm. We employed the TVAL3 CS reconstruction algorithm [16].

Firstly, experiments involving non-negative measurement,
non-negative measurement combined with mean subtraction, and complementary measurement were performed; the reconstruction results are shown in Fig. 2(a–c), respectively, for comparison. For further comparison, the original spectrum was obtained using a CCD and is shown in Fig. 2(d). The PMT counting period corresponding to one DMD action was 10 ms in all cases. The sampling ratio (the ratio between the number of measurements and the length of the spectral signal) was 30% for the first two cases, while the total sampling ratio of complementary measurement (including both positive and negative measurements) was also 30%. From Fig. 2, it is apparent that the spectral CS reconstruction quality of the non-negative measurement is very poor. The non-negative measurement combined with the mean subtraction method yields slightly improved spectral reconstruction quality. However, this improvement is limited and unsatisfactory. In contrast, the reconstruction quality of the complementary mode is far superior to that of the non-negative mode.

Secondly, we investigated the influence of the sampling ratio on the spectral reconstruction quality of the complementary mode. The selected sampling ratios were 10%, 20% and 30% and the results are shown in Fig. 3(a–c) respectively. The integration times (PMT counting period) for different sampling ratio were all 10 ms. From Fig. 3, it is apparent that the reconstruction quality improves as the sampling ratio increases. It can also be seen that the quality of the 20% sampling ratio of the complementary mode is superior to that of non-negativity or mean subtraction mode within 30% sampling ratio (Fig. 2(a and b)). This means that the complementary mode can yield superior reconstruction quality within a shorter sampling time. If we adopt a double-arm structure in the complementary mode, the total sampling time can be further decreased by half, as explained above.

Thirdly, we investigated the influence of the integration time on the spectral reconstruction quality of the complementary mode. Integration times of 1, 5, and 10 ms were selected and the results are shown in Fig. 4(a–c), respectively. The sampling ratio was 30% in all cases. From Fig. 4, it is apparent that the reconstruction quality improves with increased integration time. This is because the signal is increased as the PMT integration time increases, while the noise is averaged. Therefore, the reconstruction quality improved.

6. Discussions

From the experiments in Section 2, it is apparent that higher sampling ratios and longer integration times yield improved recovery quality; however, this increases the total measurement time. It is generally accepted that longer integration time equals to reduce the noise in the measurement; thus, this is favorable for the realization of high-quality recovery. However, it is not clear whether this is also the case for the sampling ratio.

To examine this behavior, we performed a simulation experiment, in which a full complementary compressive sampling and recovery process were simulated for a virtual spectral signal. To evaluate the recovery quality, we calculated the mean square error (MSE) of the recovered spectral signal and compared it to the original signal. Gaussian noise was added to both the positive and negative measurement data and both the mean and standard variance of noise were 10% the measured value. We then calculated the different MSEs obtained for various sampling ratios; the results are plotted in Fig. 5.

From Fig. 5, it is clear that the MSE is gradually reduced with increased sampling ratio; thus, the recovery quality is improved. For sampling ratio greater than 30%, the MSE remains stable but exhibits small fluctuations; this indicates that the recovery quality is not essentially improved once the sampling ratio exceeds 30%. When the sampling ratio is less than 30%, the MSE value exhibits large fluctuations; the reason for this is that the recovery exhibits some randomness and is, therefore, unreliable. Thus, a 30% sampling ratio is appropriate for use in complementary CS recovery procedures.

7. Conclusion

In this paper, we present a single-pixel complementary compressive sampling spectrometer, the performance of which is validated through experiment. The results of the evaluation experiments allow us to safely conclude that complementary measurement can yield significantly higher-quality spectral CS recovery than the conventional single-pixel spectroscopy technique within a shorter sampling time. Analysis of the relationship between the recovery quality, the sampling ratio, and the integration time indicates that a 30% sampling ratio is appropriate for effective spectral recovery, and that longer integration times are preferable.

Although our experiments are conducted at the visible wavelength, the structure can be easily extended to the IR range. This design requires the use of a single detector only; therefore, this structure is extremely appropriate for specialized scenarios, for example the weak light or the non-visible wavelength situations, in which exotic or expensive detectors are required. Thus, our design has huge potential application value.

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