Unified Huynen Phenomenological Decomposition of Radar Targets and Its Classification Applications

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Abstract—Huynen decomposition (HD) as the first formalized target decomposition has not been widely accepted. The preference for symmetry and regularity restricts not only its application but also its unification with other target dichotomies. The nonuniqueness issue then arises because we may have different dichotomies of radar targets, but we have no idea how to select them. In this paper, a unified Huynen dichotomy is developed by extending HD for a full preference for symmetry and regularity, nonsymmetry, irregularity, and their couplings. It covers all of the existing dichotomies and provides a unified selection mechanism for them. Scattering preference is identified as a main feature of target dichotomy, and its concise description is devised by relating each dichotomy to a canonical scattering. A scattering degree of preference (SDoP) parameter is defined to measure the preference of each dichotomy. In virtue of an adaptive combination and permutation of SDoPs, a scattering pyramid description of the mixed scattering is developed, which has better discrimination of target than entropy/alpha. An SDoP/alpha classification is further proposed by statistical modeling of the unified dichotomy, which is a competent alternative to entropy/alpha. The excellent performance of unified dichotomy makes us believe that the existing concerns on HD are well treated and the Huynen–Cloude controversy, in a sense, may be ended.

Index Terms—Huynen decomposition (HD), radar polarimetry, target decomposition, target extraction, unsupervised classification.

NOMENCLATURE AND ABBREVIATIONS

<table>
<thead>
<tr>
<th>Acronym</th>
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<tr>
<td>HD</td>
<td>Huynen decomposition.</td>
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<td>CD</td>
<td>Cloude decomposition.</td>
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<td>BHD</td>
<td>Barnes–Holm decomposition.</td>
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<td>YD</td>
<td>Yang decomposition.</td>
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<td>CHD</td>
<td>Canonical Huynen dichotomy.</td>
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<td>UHD</td>
<td>Unified Huynen dichotomy.</td>
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<td>HTD</td>
<td>Huynen-type target dichotomy/dichotomies.</td>
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<td>NS</td>
<td>Nonsymmetric/nonsymmetry.</td>
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<td>IR</td>
<td>Irregular/irregularity.</td>
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<td>SR</td>
<td>Symmetric and regular/symmetry and regularity.</td>
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RLF<sub>n</sub> Refined Lee filter with n × n aligned window.

Di<sub>i</sub> <i>i</i>th (i = 1, 2, ..., 9) target dichotomy in UHD.

Unitary matrices accounting for antenna roll transform, ellipticity transform, and absolute phase transform.

λ<sub>j</sub> and u<sub>j</sub> jth (j = 1, 2, 3) eigenvalue and eigenvector of mixed target <i>T</i>.

<sub>Scd</sub> Dominant single target and its Pauli vector extracted by CD from <i>T</i>.

Equivalent single target and remnant N-target extracted by <i>Di</i> from <i>T</i>.

Total power and Pauli vector of <i>T<sub>Si</sub></i>.

Scattering degree of preference.

Preferred vector and SDoP of <i>Di</i>.

Average SDoP parameters of CHD and UHD.

SDoP for canonical surface, dihedral, and volume scatterers.

Cloude–Pottier alpha angle and the developed Li–Zhang alpha angle.

Entropy/alpha classification and the proposed SDoP/alpha classification.

I. INTRODUCTION

The concept of target decomposition was first formalized by Huynen in 1970. In his Ph.D. dissertation on “Phenomenological Theory of Radar Targets” [1], Huynen not only demonstrated that radar targets can be decomposed like waves but also indicated that polarimetric decomposition is a feasible method for understanding complex targets. Pioneered by this work, there have been many other decomposition techniques developed hitherto, and intense attentions have been paid to this field in the past four decades. Some comprehensive reviews on the existing target decomposition approaches were presented in [2]–[4].

Although of theoretical importance, Huynen decomposition (HD) has not received wide attentions and applications. It is thought to be one of the main visionary concepts of Huynen that is not widely accepted today [5], [6]. HD has been mentioned...
often in the literatures because it is the first decomposition or it is a typical target dichotomy. However, only a minority of the reviews contributed a particular introduction to or focus on the HD [2–4, 6–24]. Three main factors impacting the application of HD are summarized here. The first one arises from the finding of Barnes and Holm that HD is not unique because there are two other dichotomies that possess the same roll-invariance around the line of radar sight [8, 9]. Barnes–Holm decomposition (BHD) relaxes Huynen’s preference for symmetry and regularity (SR) [25], but it was criticized for having little insight into the physics of scattering, because it creates two “exotic” worlds with imbalanced preferences for left or right helices [17, 26]. The second one comes from the Huynen–Cloude controversy in 1992, as simply depicted in [17] and [27]. Cloude pointed out that HD cannot provide the “global” invariance, i.e., an invariance under all unitary transforms, and mathematically, there are infinite target dichotomies if Huynen’s restriction on roll-invariance is removed, but the eigenvector-based Cloud decomposition (CD) can ensure the satisfactory result [2, 4, 10, 11]. He hence concluded that there is no target dichotomy but only one unique decomposition, i.e., CD [10]. Huynen argued that the physical significance of CD is lost in the conduct of eigendecomposition, and the related parameter proliferation problem makes it very “dubious,” but HD has clear physical significance because it caters to the world of basic symmetry in which we live [26, 28]. He concluded that the only decomposition that corresponds to the real world of symmetry is HD [26]. Huynen’s justification for HD received the support from Pottier [6] and Holm [7, 29]. However, Cloude’s concerns of HD gained more followers [3, 24, 30–33], which have greatly impacted the use of HD [15, 24, 34–37]. The last one was contributed by Yang et al. with the finding that HD cannot extract a desired target because it is not always stable [13, 14]; thus, Yang et al. proposed a modification to the HD. However, the first two factors on HD are so prominent that Yang decomposition (YD) received only limited notice [3, 22].

Huynen’s concerns of CD have been properly treated: The success of entropy/alpha classification reveals its physical significance [38], and the lossless and sufficient roll-invariant decomposition solves the parameter proliferation problem [39]. However, the concerns on HD, mainly regarding the nonuniqueness issue, have not been handled hitherto. This issue showed itself in two forms in the existing literatures. The first one is there may mathematically exist infinite ways to decompose a mixed target into the sum of a single target and an $N$-target, but the preference for SR reduces them to HD only. We do not think that such nonuniqueness of HD is worthy of our particular attention because it also exists in other decompositions. Take CD for example, as indicated by Cloude et al. [40], there are infinite ways to decompose a mixed target into the sum of three single targets, but orthogonality restricts the infinity to only Cloude’s. In this sense, the uniqueness controversy between Huynen and Cloude is trivial because they held different views on how to treat the same problem. The second one arises since BHD and YD can also conduct target dichotomy like HD. They together provide us three different points of view: Huynen’s preference for SR, Barnes–Holm’s concentration on roll-invariance, and Yang’s concerns of stability. The dichotomy realities of the three views are all reasonable, but a common consciousness has not been reached. We thus have no idea on how to select them, especially between BHD and YD. Such nonuniqueness is the underlying reason restricting the application of Huynen-type target dichotomies (HTD). A good way to solve this is to generalize these dichotomies to a unified Huynen dichotomy (UHD) and devise a fair mechanism to select the most appropriate one, which is detailed in Section II.

As for the concern that HD cannot provide a “global” invariance, we find that it is not always necessary. This is a specific attribute of CD, based on which Cloude and Pottier devised the entropy/alpha classification in response to Huynen’s concerns of the physical significance of CD. Clearly, HTD only enables a “local” invariance, i.e., the invariance under a certain unitary transform. Nevertheless, such local invariance gives each HTD a specific attribute, the scattering preference, a term that we use to depict the observations that certain target scattering information is preserved in the extracted single target by each HTD. As for HD, Huynen qualitatively explained its preference using the three phenomenological characters, i.e., SR, nonsymmetry (NS), and irregularity (IR). Although having practical relevance, it is not clear how useful the description is for a detailed interpretation of the data [41]. We also find that the information that this description conveys is limited. A novel description of scattering preference is thus developed in Section III by directly relating each HTD to a canonical scattering. By this means, we can see that HTD is born with physical significance.

An excellent decomposition should not only be able to extract the single target but also be able to characterize and describe the mixed target, which can no longer be represented by a dominant single target. It was stated in [24] and [35] that the performance of HD deteriorates as the entropy increases, i.e., it cannot be used to analyze the random mixed target as often that occurs in natural scenes. This, in fact, also exists in CD, but it was successfully solved by Cloude and Pottier using the concept of average target. To advance the application of UHD, a scattering pyramid scheme is devised in Section V by adaptively permutating and combining several HTD of different preferences to describe the scattering of mixed target. Comparative experiments with entropy/alpha overturn the general impression that HTD is inferior to CD.

Another task of this paper is dedicated to investigate the potential consistency between HTD and CD, which is conducted in two aspects. The first one arises in Section IV on the extraction of dominant single targets. A mathematical convergence to CD is clearly displayed by extending HD to the canonical Huynen dichotomy (CHD, which provides a simplification to YD, as indicated in the Appendix) and by further extending BHD and CHD to UHD. UHD can obtain very consistent extractions as CD. The second one arises in Section VI by using UHD to further model the mixed target scattering based on the average target concept of Cloude–Pottier, which enables a scattering degree of preference (SDoP)/alpha classification and is demonstrated to be a competent alternative to entropy/alpha. The effects of target orientation and speckle filtering on the classifications are also evaluated and discussed in Sections V-B and VI-C, as well as in Section VII.
Fig. 1. Phenomenological significance of the nine Huynen parameters.

HD was once treated as a decomposition procedure without much practical value [15]. This paper is dedicated to unify the existing HTD and to apply them to the advanced classification applications. Our work makes us believe that the concerns on HD have been well treated, that the performance of HTD has been well established, and that the Huynen–Claude controversy may be ended. This paper is concluded in Section VIII.

II. UNIFICATION OF HTD

A. CHD

HD is built on the physical realizability conditions of Kennaugh matrix $K$, and these conditions can be concisely expressed by the target coherence matrix $T$. If the scattering of the target through the reciprocal propagation medium is measured by a monostatic radar, then the matrix $T$ can be expressed as

$$
T = \langle \mathbf{k} \cdot \mathbf{k}^H \rangle = \begin{bmatrix}
T_{11} & T_{12} & T_{13} \\
T_{21} & T_{22} & T_{23} \\
T_{31} & T_{32} & T_{33}
\end{bmatrix} = \begin{bmatrix}
2A_0 & C - jD & H + jG \\
C + jD & B_0 + B & E + jF \\
H - jG & E - jF & B_0 - B
\end{bmatrix}
$$

(1)

where $\langle \cdot \cdot \cdot \rangle$ is the operation of ensemble average, $k$ is the Pauli vector, and superscript $H$ indicates the conjugate transpose. The parameters at the right side are termed as the Huynen parameters because Huynen first bridged them to the phenomenological characters of radar targets [42]. As shown in Fig. 1, $2A_0$, $B_0 + B$, and $B_0 - B$ are the generators of SR, IR, and NS, respectively, while the others denote the pairwise couplings between the generators.

As for the mixed target subjected to spatial and/or temporal variations, the matrix $T$ is positive semidefinite; hence, the three second-order principal minors should be nonnegative

$$
\begin{align*}
B_0^2 &> B^2 + E^2 + F^2 &\quad &-1) \\
2A_0(B_0 - B) &> H^2 + G^2 &\quad &-2) \\
2A_0(B_0 + B) &> C^2 + D^2 &\quad &-3).
\end{align*}
$$

(2)

For mixed target, $B_0^2 > B^2 + E^2 + F^2$ (1)

Matrix $T$ turns to rank-one for single target. Nine equations are then obtained, but only four of them are independent because a single target has five degrees of freedom. We can therefore extract the following three equation groups:

$$
\begin{align*}
2A_0(B_0 - B) &= H^2 + G^2 \\
2A_0(B_0 + B) &= C^2 + D^2 \\
2A_0(E + jF) &= (C + jD)(H + jG) \\
(B_0 + B) \cdot 2A_0 &= C^2 + D^2 \\
(B_0 + B)(B_0 - B) &= E^2 + F^2 \\
(B_0 + B)(H + jG) &= (C - jD)(E + jF) \\
(B_0 - B) \cdot 2A_0 &= H^2 + G^2 \\
(B_0 - B)(B_0 + B) &= E^2 + F^2 \\
(B_0 - B)(C - jD) &= (H + jG)(E - jF)
\end{align*}
$$

(3)

Each group of (3) is self-contained, and from them, we get the following three equations, respectively:

For single target,

$$
\begin{align*}
2A_0(B_0 - B) &= H^2 + G^2 \\
2A_0(B_0 + B) &= C^2 + D^2 \\
2A_0(E + jF) &= (C + jD)(H + jG)
\end{align*}
$$

(4)

Equations (2-1) and (4-1) reveal the different behaviors of parameters $(B_0, B, E, F)$ in the two target scenarios, which inspired Huynen using a target dichotomy to decompose the mixed target $T$ into the sum of an equivalent single target $T_S$ and a remnant $N$-target $T_N$ by analogy to the wave dichotomy [1]

$$
T = T_S + T_N = \begin{bmatrix}
2A_0 & C - jD & H + jG \\
C + jD & B_0 + B & E + jF \\
H - jG & E - jF & B_0 - B
\end{bmatrix}
$$

+ \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix} = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
$$

(5)

By replacing $(B_0, B, E, F)$ with $(B_0S, B_0S, B_0E, B_0F)$, the decomposed parameters $(B_0S, B_S, E_S, F_S)$ can be inverted from (3-1) based on preserved parameters $(2A_0, C, D, G, H)$, which are completely kept in $T_S$ because Huynen insisted that the real world physically prefers SR and the related scattering information should not appear in $T_N$ [28]. This kind of world view restricts the applicability of HD to the ideal SR scatterer only, i.e., HD fails to analyze the complex IR and NS scatterers. This has been independently validated by Barnes and Holm [9], Yang et al. [14], and Paladini [17] on the decomposition of a distributed dihedrally scattered. As a response, Huynen indicated that our attention should turn from $T_S$ to $T_N$ in these cases [26]. However, HD decomposes the IR- and NS-related parameters $(B_0, B, E, F)$ into both $T_N$ and $T_S$. The attentions on $T_S$ or $T_N$ are all inappropriate for integrated characterization of such radar targets.

Hence, our first attempt is to extend HD to the preferences for IR and NS. This can be simply achieved because we still have two mixed target inequations in (2) with their corresponding
single target equations appearing in (4). As for parameters \(2A_0, B_0 - B, H, G\), if we define
\[
2A_0 = B_0' - B', \quad B_0 + B = 2A_0', \quad B_0 - B = B_0' + B' \quad (6)
\]
which enables us to write (2-2) and (4-2) into the similar forms to (2-1) and (4-1), respectively
\[
\begin{align*}
B_0'^2 &= B_0'^2 + H^2 + G^2 & \text{for single target} \\
B_0''^2 > B_0'^2 + H^2 + G^2 & \text{for mixed target. (7)}
\end{align*}
\]
Analogous to HD, we then have
\[
T = T_S' + T_N' = \begin{bmatrix} 2A_{0S} & C - jD & H + jG \\ C + jD & B_0 + B & E + jF \\ H - jG & E - jF & B_{0S} - B_S \end{bmatrix}
\]
\[
+ \begin{bmatrix} 2A_{0N} & 0 & H_{N} + jG_{N} \\ 0 & 0 & 0 \\ H_{N} - jG_{N} & 0 & B_{0N} - B_{N} \end{bmatrix}
\quad (8)
\]
Dichotomy (8) prefers IR because it preserves the IR-related Huynen parameters \((B_0 + B, C, D, E, F)\) into the matrix of single target. Based on these preserved parameters, the decomposed parameters \((2A_{0S}, B_{0S} - B_S, G_S, H_S)\) can be easily retrieved from (3-2).

Likewise, as for parameters \((2A_0, B_0 + B, C, D)\), if we define
\[
2A_0 = B_0'' - B''_0, \quad B_0 + B = B_0'' + B''_0, \quad B_0 - B = 2A_0' \quad (9)
\]
from (2-3) and (4-3), we then have
\[
\begin{align*}
B_0'^2 &= B_0'^2 + C^2 + D^2 & \text{for single target} \\
B_0''^2 > B_0'^2 + C^2 + D^2 & \text{for mixed target. (10)}
\end{align*}
\]
Hence, we can also obtain the following target dichotomy:
\[
T = T_S'' + T_N'' = \begin{bmatrix} 2A_{0S} & C_S - jD_S & H + jG_S \\ C_S + jD_S & B_{0S} + B_S & E + jF_S \\ H - jG_S & E - jF_S & B_0 - B \end{bmatrix}
\]
\[
+ \begin{bmatrix} 2A_{0N} & C_N - jD_N & 0 \\ C_N + jD_N & B_{0N} + B_N & 0 \\ 0 & 0 & 0 \end{bmatrix}
\quad (11)
\]
Dichotomy (11) prefers NS because it preserves the NS-related parameters \((B_0 - B, E, F, G, H)\) into the single target coherence matrix, and the decomposed parameters \((2A_{0S}, B_{0S} + B_S, C_S, D_S)\) can be obtained from (3-3) based on the preserved parameters.

Dichotomies (8) and (11) provide two useful supplements to HD, and they together offer three dichotomies preferring SR, IR, and NS, respectively. To unify them, a fair mechanism should be devised so that each of them has a chance to be selected. Here, our strategy is based on the total power (SPAN) of the extracted single target: Use the three dichotomies to extract \(T_S, T_S',\) and \(T_S''\) from \(T,'\) and select the one with the maximum SPAN as the final extraction. We name this decomposition the CHD because Huynen has claimed the existences of \(T_N'\) and \(T_N''.\) They were termed as the type I and type II symmetrical canonical targets but were abandoned for having no such natural appeal as \(T_N\) [1].

Yang et al. indicated that HD cannot stably extract a desired single target when the \(2A_0\) parameter of matrix \(K\) is small or even zero [14]. They proposed a modified HD based on the \(K\) matrix by constructing two simple transforms of matrix \(K\) for two new matrices \(K_1\) and \(K_2,\) then identified \(K, K_1,\) or \(K_2\) whose \(2A_0\) parameter is the maximum, and used HD to decompose it for the final dichotomy. YD greatly improves the stability of HD but has not attracted much notice because its physical significance is somewhat unclear. Since \(2A_0\) is the generator of SR, the instability of HD in small \(2A_0\) case is just because HD is not applicable to SR subordinate targets. As formulated in the Appendix, the \(2A_0\) parameters of \(K_1\) and \(K_2\) are \(B_0 + B\) and \(B_0 - B,\) which are, respectively, the generators of IR and NS; thus, the adaptive selection in YD is to judge the dominant character of the target being SR, IR, or NS. It is also shown in the Appendix that the \(K_1\)- and \(K_2\)-related decompositions are just dichotomies (8) and (11), except that CHD identifies the final dichotomy based on SPAN instead of \(2A_0.\) CHD thus simplifies YD and reveals its physical significance from a different starting point.

B. UHD

Besides the physical meaning, Barnes and Holm found that the \(N\)-target \(T_N\) in (5) is mathematically invariant to the antenna roll transform of \(U_\varphi\) [8], [9]. Given vector \(q\) belonging to the null space of \(T_N,\) we can have
\[
T_N q = 0 \Rightarrow T_N(\varphi)q = U_\varphi T_N U_\varphi^H q = 0
\]
with \(U_\varphi = \begin{bmatrix} 1 & 0 \\ 0 & \cos 2\varphi & \sin 2\varphi \\ 0 & -\sin 2\varphi & \cos 2\varphi \end{bmatrix} \quad (12)
\]
Then, the Pauli vector \(k_S\) of the equivalent single target \(T_S\) can be reconstructed by [9]
\[
T q = (T_S + T_N) q = T_S q = k_S \cdot T_S q \quad (13)
\]
Equation (12) indicates that \(q\) is the eigenvector of \(U_\varphi,\) as listed in Table I. Hence, we can obtain three estimations of \(k_S\) under the roll-invariance of \(T_N\) [9]
\[
\begin{align*}
k_{s1} &= \frac{1}{\sqrt{2A_0}} \begin{bmatrix} 2A_0 \\ C + jD \\ H - jG \end{bmatrix} \\
k_{s2} &= \frac{1}{\sqrt{2(B_0 + F)}} \begin{bmatrix} C - G + jH - jD \\ B_0 + B + F - jE \\ E + jB_0 - jB - jF \end{bmatrix} \\
k_{s3} &= \frac{1}{\sqrt{2(B_0 + F)}} \begin{bmatrix} C + G - jH - jD \\ B_0 + B + F - jE \\ E - jB_0 + jB - jF \end{bmatrix}
\end{align*}
\quad (14)
\]
where \(k_{s1}\) just corresponds to the single target extracted by HD, and the dichotomies corresponding to \(k_{s2}\) and \(k_{s3}\) are, respectively, BHD I and II.

The roll-invariance condition in BHD relaxes the SR preference of HD and indicates two other dichotomies. The preference of BHD can be likewise inferred. The introduction of \(q_2\) and \(q_3\) (in Table I) into \(T q = T S q\) of (13) shows the reservation of the combinations of the second and third columns of \(T,\) which
reveals BHD’s preference for the coupling of IR and NS, because these two columns physically relate to IR and NS, respectively, as illustrated in Fig. 1. This is necessary in the description of the helix-like scatterer. Likewise, we can also have the coupling of SR and IR, as well as that of SR and NS. Our second attempt is thus to unify CHD and BHD by generalizing CHD to the preferences for the pairwise couplings of SR, IR, and NS. This can be easily achieved because $T''$ and $T''_N$ are found invariant to the following two unitary transforms, respectively:

$$U_\tau = \begin{bmatrix} \cos 2\tau & 0 & j \sin 2\tau \\ 0 & 1 & 0 \\ j \sin 2\tau & 0 & \cos 2\tau \end{bmatrix}$$

$$U_\alpha = \begin{bmatrix} \cos 2\alpha & j \sin 2\alpha & 0 \\ j \sin 2\alpha & \cos 2\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where $U_\tau$ refers to the ellipticity transform, such as if we replace a linearly polarized antenna with a circularly polarized antenna. $U_\alpha$ corresponds to the absolute phase transform that resulted from the slant range variation or from the change of antenna phase center. From the invariance of $T''_N$, we obtain that

$$T''_N q = 0 \Rightarrow T''_N(\tau)q = U_\tau T''_N U^\tau q = 0 \quad (16)$$

which signifies that $q$ is the eigenvector of $U_\tau$. Analogous to BHD, we obtain the dichotomies

$$k_{S4} = \frac{1}{\sqrt{B_0 + B}} \begin{bmatrix} C - jD \\ B_0 + B \\ E - jF \end{bmatrix}$$

$$k_{S5} = \frac{1}{\sqrt{2A_0 + B_0 + B + 2H}} \begin{bmatrix} 2A_0 + H + jG \\ C + E + jD + jF \\ B_0 - B + H - jG \end{bmatrix}$$

$$k_{S6} = \frac{1}{\sqrt{2A_0 + B_0 + B - 2H}} \begin{bmatrix} 2A_0 - H - jG \\ C - E + jD - jF \\ -B_0 + B + H + jG \end{bmatrix}$$

where the vector $k_{S4}$ just corresponds to dichotomy (8).

Likewise, the invariance of $T''_N$ indicates that $q$ is also the eigenvector of $U_\alpha$. Hence, we further obtain

$$k_{S7} = \frac{1}{\sqrt{B_0 - B_0}} \begin{bmatrix} H + jG \\ E + jF \\ B_0 - B \end{bmatrix}$$

$$k_{S8} = \frac{1}{\sqrt{2A_0 + B_0 + B + 2C}} \begin{bmatrix} 2A_0 + C - jD \\ B_0 + B + C + jD \\ E + H - jF - jG \end{bmatrix}$$

$$k_{S9} = \frac{1}{\sqrt{2A_0 + B_0 + B - 2C}} \begin{bmatrix} 2A_0 - C + jD \\ -B_0 - B + C + jD \\ -E + H + jF - jG \end{bmatrix}$$

where the vector $k_{S7}$ just corresponds to dichotomy (11). By combining (14), (17), and (18), we finally have nine dichotomies based on the invariance of $N$-targets of CHD. For expressing conveniently, we denote the dichotomy corresponding to vector $k_{S_i}$ as $Di$ ($i = 1, 2, \ldots, 9$). Utilizing the analysis method applied to BHD, the preferences of D5 and D6 for the coupling of SR and NS, as well as the preferences of D8 and D9 for the coupling of SR and IR, can be likewise inferred. To unify the nine dichotomies, we take an adaptive strategy to select the dichotomy among the nine whose single target has the maximum SPAN. We call this decomposition the UHD because it covers all of the existing HTD such as HD (D1), BHD (D2 and D3), and YD (D4 and D7) and provides an adaptive dichotomy for all of the mixed radar targets dominated by SR, IR, NS, or their couplings.

### III. Scattering Preference Analysis of UHD

The nine dichotomies in UHD can be generally expressed as

$$T = T_{Si} + T_{Ni} = k_{Si} \cdot q_{Si}^\dagger + T_{Ni},$$

$$k_{Si} = \frac{Tq_i}{q_i^\dagger Tq_i}, \quad i = 1, 2, \ldots, 9 \quad (19)$$

where $T_{Si}$ denotes the single target extracted by $Di$ with the Pauli vector $k_{Si}$, $q_{i}$ is the eigenvector of $U_\varphi$, $U_\tau$, and $U_\alpha$, as
listed in Table I, and it satisfies

$$\mathbf{T}_q = \mathbf{T}_{Si} q_i. \quad (20)$$

Here, the significance of (20) is underlined for it mathematically shows an important characteristic of HTD, i.e., $\mathbf{D}_i$ intact reserves certain target information into $\mathbf{T}_{Si}$. We term this reservation as scattering preference, and following Huynen’s convention, we qualitatively describe it in terms of the phenomenological characters SR, IR, and NS, as listed in Table I. Although having practical relevance, it is not clear how useful the description is for a detailed interpretation of data [41]. We also find that the information that this description conveys is limited. Taking the two BHDs for example, their preferences are expressed as for the two couplings of IR and NS following this description, but we cannot further tell their difference. To advance the application of UHD, a better description of the scattering preference information is necessary.

In fact, (20) can provide further information if we transform it into the following form:

$$q_i^R^H \mathbf{T}_q = q_i^R^H \mathbf{T}_{Si} q_i = |q_i^R^H k_{Si}|^2. \quad (21)$$

If $q_i$ is treated as a Pauli vector corresponding to the canonical scatterers, such as the surface, helix, dihedral, dipole, and volume scatterers, as listed in Table I, then the left side of (21) just indicates the scattering similarity between $q_i$ and the random scatterer $\mathbf{T}$ [43], and the right side shows the similarity between $q_i$ and the deterministic scatterer $k_{Si}$ [44]. The scattering similarity with $q_i$ is thus reserved by $\mathbf{D}_i$, and we infer that $\mathbf{D}_i$ naturally prefers scattering $q_i$. This description directly bridges $\mathbf{D}_i$ to a canonical scattering, from which we can immediately attach the preferences of $\mathbf{D}_2$ and $\mathbf{D}_3$ to the left- and right-wound helices, respectively; they are consistent with Huynen’s comments on BHD from the viewpoint of power received by an antenna [26]. However, it should be noted that the preferred scattering $q_i$ is just a physical description of the mathematical preservation of $\mathbf{D}_i$; it cannot replace $k_{Si}$ to determine the scattering of the extracted single target. As formulated in (19), only when $q_i$ is the eigenvector of $\mathbf{T}$, then $q_i$ and $k_{Si}$ correspond to the same scattering. Hence, although $q_5$, $q_9$, $q_8$, and $q_0$ correspond to different rotated dipoles, it does not mean that $k_{SS}$, $k_{SS}$, $k_{SS}$, and $k_{SS}$ are also canonical dipole if $\mathbf{T}$ is deoriented beforehand.

In addition to the qualitative description in terms of canonical target scattering, we further define an SDoP parameter to quantitatively evaluate the preference degree of $\mathbf{D}_i$

$$\text{SDoP}_{\mathbf{D}_i} = \frac{\text{SPAN}_{\mathbf{S}_{Si}}}{\text{SPAN}} = \frac{\|k_{Si}\|^2_F}{\sum_{j=1}^3 T_{jj}^2} \cdot \frac{\|T_q\|^2_F}{\sum_{j=1}^3 T_{jj}^3} \cdot \left(\frac{q_i^R^H q_i}{\|q_i^R^H q_i\|} \right)^3 \sum_{j=1}^3 T_{jj} \quad i = 1, 2, \ldots, 9 \quad (22)$$

where $\text{SPAN}_{\mathbf{S}_{Si}}$ and $\text{SPAN}$ denote the power of single target $T_{Si}$ and mixed target $\mathbf{T}$, respectively. SDoP thus measures the relative power and is comparable to the wave degree of polarization which measures the relative power between the fully polarized part and the whole of the wave. As a key feature of HTD, SDoP will be used in Sections V and VI to provide us two novel classifications of radar targets.

UHD is applied to processing the four-look NASA/JPL L-band AIRSAR San Francisco data in the following, which covers a variety of scatterers, such as ocean, urban area, and vegetation. Fig. 2(l) gives the Pauli image of the original data, and Fig. 2(a)–(i) exhibits the Pauli images of the single targets extracted by $\mathbf{D}_i$, respectively. The dark blue in Fig. 2(a) reflects the preference of $\mathbf{D}_1$, i.e., HD for the SR scatterers like rough ocean surface. Fig. 2(d) gives the extraction of $\mathbf{D}_4$. The wide distribution of red color shows its preference for the IR dihedral scatterers. The distributed dihedral scatterer utilized by Barnes and Holm [9], Yang et al. [14], and Paladini [17] to indicate the failure of HD, however, can be successfully decomposed by $\mathbf{D}_4$. This dichotomy is thus useful for building detection because buildings generally contribute to the most dihedral scattering in the urban area. $\mathbf{D}_7$ prefers the NS volume scatterers, and the green forests in Fig. 2(g) clearly reflect this; thus, it can be utilized for forest detection. The images of other six dichotomies prefer the pairwise hybrids of red, green, and blue. Taking Fig. 2(b) and (c) for examples, they prefer red and green because the combinations of the second and third columns of $\mathbf{T}$ are reserved by $\mathbf{D}_2$ and $\mathbf{D}_3$. However, it is difficult to visually relate them with the real targets because the helix scatterings that they preferred are tiny in this scene. The preferences of $\mathbf{D}_5$ and $\mathbf{D}_6$, as well as the preferences of $\mathbf{D}_8$ and $\mathbf{D}_9$, can be likewise validated. Target extractions on the Alcatraz Island, framed in Fig. 2(l), are detailed in Fig. 3 further. All of these clearly illustrate the scattering preference of $\mathbf{D}_i$. UHD selects the most preferable extraction of the nine dichotomies as the final single target extraction. Figs. 2(j) and 3(j) illustrate the UHD extractions on the two scenes, respectively, which are much closer to the original mixed scattering than the extraction of $\mathbf{D}_i$. Furthermore, UHD makes the target scattering signature much clear by adaptively filtering the unwanted noisy scatterings. The island scattering in Fig. 3(j) is strengthened because the “blur cover” in Fig. 3(l) is removed, which improves the capability for target identification. By counting the contribution of $\mathbf{D}_i$ to the final extraction, UHD also provides a coarse classification of terrain. The SR scatterer is found covering only 41% of the San Francisco scene, i.e., the majority of the scene is inappropriate for HD. This validates the restricted application of HD in a sense.

IV. MATHEMATICAL EVALUATION OF UHD

The mixed target coherence matrix $\mathbf{T}$ can be eigendecomposed as follows:

$$\mathbf{T} = \sum_{j=1}^3 \sqrt{\lambda_j} \mathbf{u}_j \cdot \sqrt{\lambda_j} \mathbf{u}_j^H, \quad \lambda_1 \geq \lambda_2 \geq \lambda_3 \geq 0$$

$$\mathbf{T}_{\text{Scd}} = k_{\text{Scd}} \cdot k_{\text{Scd}}^H \mathbf{u}_1 \cdot \mathbf{u}_1^H + k_{\text{Scd}} \cdot k_{\text{Scd}}^H \mathbf{u}_2 \cdot \mathbf{u}_2^H \quad (23)$$

where $\lambda_j$ is the eigenvalue associated with the eigenvector $\mathbf{u}_j$. Cloude interpreted $\mathbf{u}_j$ as a single target and chose $\mathbf{u}_1$ as the dominant scattering because it is the optimal estimation to the unit target vector [10], [38]. $\mathbf{T}_{\text{Scd}}$ in (23) denotes the coherence matrix of dominant scattering, and $k_{\text{Scd}}$ is its Pauli vector. CD therefore can secure an optimal scattering estimation, and the performance of UHD can be well demonstrated by comparing with it.

1The Pauli image of mixed target $\mathbf{T}$ is created by treating $\sqrt{T_{22}}$, $\sqrt{T_{53}}$, and $\sqrt{T_{11}}$ as the red, green, and blue colors, respectively. The Pauli image of single target $T_{Si}$ is obtained by using the modulus of the second, third, and first elements of Pauli vector $k_{Si}$, respectively, as red, green, and blue.
Figs. 2(k) and 3(k) display the dominant scatterings extracted by CD from scenes of San Francisco and Alcatraz Island, respectively. They look nearly the same as the UHD extractions in Figs. 2(j) and 3(j). To validate this, the Pauli vector $k_S$ extracted, respectively, by HD, BHD, CHD (YD), and UHD is compared with $k_{Scd}$ on the San Francisco scene in the following.

Fig. 4(a)–(c) shows the relationship between the moduli of $k_S(i)$ and that of $k_{Scd}(i)$, respectively, where $k_S(i)$ is the $i$th element ($i = 1, 2, 3$) of $k_S$, and the meaning of $k_{Scd}(i)$ can be likewise inferred. The relationship on the Frobenius norm (F-norm) is given in Fig. 4(d). A convergence to CD is clearly revealed from HD and BHD to CHD, and further to UHD. For quantitative demonstration, we calculate the relative residues ($rr$) between $k_S$ and $k_{Scd}$ on F-norm and moduli as follows:

\[
\begin{align*}
rr_F &= \left| \frac{\|k_S\|_F - \|k_{Scd}\|_F}{\|k_{Scd}\|_F} \right| \\
rr_i &= \left( \frac{|k_S(i) - k_{Scd}(i)|}{|k_{Scd}(i)|} \right), \quad i = 1, 2, 3.
\end{align*}
\]
In this way, a sample of $rr_1$, $rr_2$, $rr_3$, and $rr_F$ can be obtained at each image pixel. The mean and variance of all of the final $rr_1$ samples and those of $rr_2$, $rr_3$, and $rr_F$ are calculated and listed in Table II. It is indicated that HD and BHD I and II perform equivalently but quite largely deviate from CD. The deviation is compensated by CHD to some extent, but their differences are still clear. UHD reduces the residue the most, and only tiny differences are left. In particular, the average $rr_F$ between UHD and CD is merely 0.99%, and their F-norm relation in Fig. 4(d) is nearly linear. If the confidence thresholds of $rr_1$, $rr_2$, $rr_3$, and $rr_F$ are all fixed as 20%, there are then 91.73%, 91.91%, 90.20%, and 100% of pixels that give support to the consistency between UHD and CD, respectively. To remove the potential biases from the data set, the two decompositions are also compared on the DLR L-band ESAR data of Oberpfaffenhofen, and a similar convergence trend is also observed. It is exhibited that only 47.16% of the Oberpfaffenhofen scene is appropriate for HD and the average $rr_F$ is also as tiny as 1.3854%. All of these not only demonstrate UHD’s superiority over the existing HTD but also indicate the potential unifiability of UHD and CD.

UHD does not seem as straightforward as CD since it needs to perform nine dichotomies for the final target extraction. In fact, it can be achieved quickly because the nine extractions in (14), (17), and (18) are just the columns of $T$ or their pairwise combinations. Therefore, unlike CD, the eigendecomposition is avoided in UHD.
TABLE II
STATISTICAL CHARACTERS OF THE RELATIVE RESIDUE BETWEEN HTD AND CD ON THE RETRIEVAL OF DOMINANT SINGLE TARGET PAULI VECTOR

<table>
<thead>
<tr>
<th>Dichotomy</th>
<th>( r_{r_1} )</th>
<th>( r_{r_2} )</th>
<th>( r_{r_3} )</th>
<th>( r_{r_4} )</th>
<th>( r_{r_5} )</th>
<th>( r_{r_6} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Huynen</td>
<td>0.2699</td>
<td>2.0820</td>
<td>0.2294</td>
<td>0.1484</td>
<td>0.2110</td>
<td>0.8577</td>
</tr>
<tr>
<td>Barnes-Holm I</td>
<td>0.3412</td>
<td>0.1065</td>
<td>0.2643</td>
<td>0.2015</td>
<td>0.3978</td>
<td>6.5844</td>
</tr>
<tr>
<td>Barnes-Holm II</td>
<td>0.2815</td>
<td>0.1133</td>
<td>0.2559</td>
<td>20.0721</td>
<td>0.3665</td>
<td>9.5643</td>
</tr>
<tr>
<td>Canonical Huynen</td>
<td>0.1087</td>
<td>0.0466</td>
<td>0.1220</td>
<td>0.3332</td>
<td>0.1318</td>
<td>0.1713</td>
</tr>
<tr>
<td>Unified Huynen</td>
<td>0.0559</td>
<td>0.0184</td>
<td>0.0623</td>
<td>0.0312</td>
<td>0.0810</td>
<td>0.1075</td>
</tr>
</tbody>
</table>

\( r_{r_1}, r_{r_2}, \) and \( r_{r_3} \) in the 1st row denote the relative residue on the three Pauli vector components, respectively, and \( r_{r_5} \) denotes that on the F-norm of the Pauli vector.

V. UNSUPERVISED CLASSIFICATION BASED ON ADAPTIVE PERMUTATION OF UHD

Besides the application to scattering extraction, another utility of UHD lies in its evident reflection of target scattering because \( D_i \) has its own scattering preference. The preference just arises from the “misfortune” that \( D_i \) only provides an invariance under a certain unitary transform \( U_\lambda, U_\tau, \) or \( U_\alpha \), which enables us to directly relate \( D_i \) to a fixed canonical scattering \( q_i \). By replacing \( q_i \) with eigenvector \( u_j \) and substituting \( \lambda_i, u_j \) for \( T_{Si} \), we then find that CD also satisfies (20). Nevertheless, \( q_i \) is no longer stationary here but depends on \( T \) due to CD’s pursuit of the invariance under all unitary transforms. The scattering preference of CD is thus unclear.

A. Scattering Pyramid Classification Scheme

A simple classification can be achieved by directly comparing SDoP\(_{D_j}\) to identify the most preferable scattering as the dominant scattering preference of target. For instance, the target is labeled as “more preferable to surface” if SDoP\(_{D_j}\) is larger than other SDoP\(_{D_i}\). This classification has been used in Sections III and IV to indicate the restricted application of HD. It is based on a radical assumption that there is always a dominant scattering preference in the mixed scattering. This works well for low-randomness target but is no longer tenable for medium- and high-randomness targets, because these targets may have several comparable preferences and we cannot extract a significantly stronger one from them. To extend UHD for such targets, we need to devise an advanced scheme. Chen et al. [43] recently gave another interpretation to CD. They proposed to characterize the low-entropy target only using scattering \( u_1 \), to describe the medium-entropy target using both \( u_1 \) and \( u_2 \), and to represent the high-entropy one using all \( u_1, u_2, \) and \( u_3 \). However, since \( u_j \) cannot evidently reflect target scattering like \( k_{Si} \), Chen et al. thus used the similarity parameter to indicate the scattering mechanism instead. Here, this method is improved to a scattering pyramid by adaptively permuting and combining several HTD with different scattering preferences to jointly characterize the mixed target scattering. The pyramid is composed of several layers to indicate the different degrees of scattering randomness. Each layer is further subdivided into several blocks to reflect the different scattering mechanisms. A special characteristic of this scheme is that both the layers and the blocks are determined by the scattering preferences of HTD, i.e., scattering preferences can model both the scattering mechanism and the randomness. More dichotomies indicate more layers and blocks, and signify better description but more computation. An adaptive strategy is used to select the scene-related dichotomies from the nine based on some priori information. For instance, ocean, building, and forest dominate the San Francisco scene; D1, D4, and D7 hence can provide enough information on the scene due to their preferences for surface, dihedral, and volume scatterings, respectively. Here, we exemplify the scheme based on D1, D4, and D7. By bringing the Pauli vectors of the three dichotomies into (22), we obtain the SDoP for surface (SDoP\(_s\)), dihedral (SDoP\(_d\)), and volume scatterer (SDoP\(_v\)) as follows:

\[
\text{SDoP}_s = \frac{3}{11} \sum_{i=1}^{3} |T_{i1}|^2, \quad \text{SDoP}_d = \frac{3}{22} \sum_{i=1}^{3} |T_{i2}|^2, \quad \text{SDoP}_v = \frac{3}{33} \sum_{i=1}^{3} |T_{i3}|^2.
\]

The three parameters can be fast obtained because they directly relate to each column of \( T \). They then promote a three-layer scattering pyramid, as shown in Fig. 5(a). Each layer is composed of several blocks. Each block indicates a different scattering class and is expressed in a permutation or combination of SDoPs, SDoP\(_d\), and SDoP\(_v\). The number of blocks in each layer is different because each layer is designed to denote a different scattering scenario. Scenario I located in the third layer concerns only the significantly strong preference. This results in three potential permutations \( (P_1^3) \) of SDoPs, SDoP\(_d\), and SDoP\(_v\). The number of blocks in each layer is different because each layer is designed to denote a different scattering scenario. Scenario I located in the third layer concerns only the significantly strong preference. This results in three potential permutations \( (P_1^3) \) of SDoPs, SDoP\(_d\), and SDoP\(_v\). By inquiring which one is the strongest, three classes (blocks) which prefer surface (simplified as S), dihedral (simplified as D), and volume scattering (simplified as V) are obtained, respectively. Scenario II in the second layer appears if the contribution of the second strongest preference is also prominent. Six permutations \( (P_2^6) \) then arise, which signify six different scattering mechanisms. The target is indexed as “more preferable to surface and dihedral” if SDoP\(_s \geq \text{SDoP}_d > \text{SDoP}_v \) (simplified as SD). The other five classes, i.e., SV, DS, DV, VS, and VD, can be likewise inferred. Scenario III in the first layer indicates a chaotic state where the three preferences are comparable. Only one combination \( (C_3^3) \) is then obtained, and the target is wholly labeled as “random scatterer” (simplified as R). Nevertheless, a confusion emerges when SDoP\(_s = \text{SDoP}_d = \text{SDoP}_v \). When the target is fully determined, SDoP\(_{D_j}\) in (22) is equal to 1 because a single target \( K \) matrix cannot be decomposed further [1]. On the other hand, when the target is
fully noisy, $SDoP_{D1}$, is also equal with each other but changes to 1/3. Hence, both fully determined and fully random targets are labeled as R by comparing $SDoP_s$, $SDoP_d$, and $SDoP_v$. A solution to this can be obtained if the following average $SDoP$ ($SDoP_3$) is formed:

$$SDoP_3 = \frac{SDoP_s^2 + SDoP_d^2 + SDoP_v^2}{SDoP_s + SDoP_d + SDoP_v}.$$  \hspace{2cm} (26)

The parameter is 1 for single target, is 1/3 for noisy target, and resides between 1/3 and 1 for the other targets. Hence, it can measure target randomness and be used to distinguish the three scattering scenarios. A target of high $SDoP_3$ has low randomness, and a dominantly preferable scattering can represent it; this corresponds to scenario I. A target of medium $SDoP_3$ has medium randomness, and the second strongest preference is added, as depicted in scenario II. The target approaches completely random when $SDoP_3$ is close to 1/3, and then, scenario III appears. By statistically counting different scatterers on the widely used PolSAR data like San Francisco and Oberpfaffenhofen, the values of 2/5 and 2/3 are determined as the boundaries of the three scenarios, and a total of ten classes are finally obtained, as illustrated in Fig. 5(a) and summarized in Table III.

Fig. 5(c) shows the classification of the San Francisco scene. Before applying UHD, the refined Lee filter with 7×7 aligned window (RLF7) is used to suppress the speckle. The meaning of RLFn can be likewise inferred. The influences of RLFn on the classification will be investigated in Section VII. We can see that the typical targets such as the ocean, buildings, forest, avenue, beach, polo field, golf course, mountain area, the Golden Gate Bridge, the Sunset Reservoir Park, and the Alcatraz Island are all well identified.

B. Influence of Target Orientation

The same target can be made to be differently presented by a simple rotation about the line of radar sight [28]. As a result, a building may be identified as forest because orientation will increase the cross-polarized scattering [45], [46]. The coherence matrix of an oriented target can be expressed as $U \varphi^TU^\varphi$, which impacts both the target extraction in (19) and the $SDoP_{D1}$ calculation in (22). To avoid these influences, deorientation should be conducted first.

The classification of the deoriented San Francisco scene is given in Fig. 5(d). The chocolate color in Fig. 5(c) denotes DV, but it changes to the dark red, denoting DS in Fig. 5(d). To reveal the change quantitatively, Table IV lists the percentages of appearance of the ten classes with and without deorientation, respectively. The changes of DS and DV are clearly presented. They account for 19.09% and 8.61% of the scene without deorientation but turn to 27.23% and 0.66% with deorientation. The decrease of DV is 7.95%, approaching to the increase of DS of 8.14%. The increases of S and D as well as the decrease of V can be also found. Deorientation thus effectively decreases the cross-polarized scattering that resulted from the misalignment between the radar and the target, which makes the classification much closer to the ground truth.

C. Comparison With Entropy/Alpha

Cloude and Pottier described the mixed target scattering using a concept of average target [38]. They interpreted CD as three single target scatterings $u_j$ occurring with probability proportional to $\lambda_j$. An entropy parameter $H$ was used to depict target randomness, which attributes the mixed scattering into three scenarios, i.e., low entropy, medium entropy, and high
entropy. By parameterizing $u_j$ with a revised Bragg scattering model, an average scattering $u_0$ is obtained to indicate the scattering mechanism; particularly, the alpha angle ($\alpha_{cp}$) of $u_0$ can directly reflect the physical property of the target, and it was then utilized to subdivide the three $H$-scenarios into eight effective zones, as indexed by $Z_i$ ($i = 1, 2, \ldots, 8$) in Fig. 5(b). The $H/\alpha_{cp}$ classification of the San Francisco scene is shown in Fig. 5(e), which is compared with the classification in Fig. 5(d) in the following.

Our intuitive impression is the similarity, which arises from the fact that both schemes utilize the scattering randomness to coarsely differentiate the targets and divide the low randomness scenario into three classes for the scatterings of surface, dihedral, and volume. Nevertheless, their difference is also obvious, and two major differences are observed. First, the forest (dark green) and the Sunset Reservoir Park (circle 3), as well as the Golden Gate Bridge in circles 3 and 7 also appear in DS and DV, which is compared with the classification in Fig. 5(d). The good separability in Fig. 5(d) reveals that it is not always necessary to further distinguish the random target. Second, the beach area in rectangle 4 shows itself as volume scattering (green) in Fig. 5(e) but turns to SD (yellow) in Fig. 5(d). We think that the latter is more consistent with the ground truth that beach generally comprises of sand.

A similar situation is also observed in the polo field (circle 5) and the golf course (circle 6). Moreover, the Reservoir Park and the Golden Gate Bridge in circles 3 and 7 also appear as volume scattering in Fig. 5(e) but turn to DS (dark red) in Fig. 5(d). The classification in Fig. 5(d) is found more credible by further referring to the optical image and the Wishart $H/\alpha_{cp}$ classification [3] of the San Francisco scene.

The deficiency of $H/\alpha_{cp}$ just originates from its statistical average modeling of mixed scattering. It can represent the mixed scattering in a total sense but also brings in certain obscurity. To explain this, a quantitative overlapping evaluation of the two schemes on the division of medium-entropy scenario II is further conducted by surveying how many scenes of Z3, Z4, and Z5 also appear in SD, DV, DS, VS, and SDV of the scattering pyramid. As listed in Table V, Z5 appears in both SD and SV; these two classes not only reveal the importance of surface scattering like Z5 but also identify the secondary preferable scattering. The extensive distribution of Z3 in DS and SD is also observed, which also provides more scattering information besides attaching the importance to dihedral scattering like Z3. The clearest illustration of the obscurity of $H/\alpha_{cp}$ is found in Z4, which characterizes the volume scattering. Nevertheless, Table V indicates that only a limited scene of Z4 corresponds to the volume-scattering-related VS and SV. The majority of Z4 emerges in DS and SD, which denote more preferable to surface and dihedral scatterings than to volume scattering. The $\alpha$ angles of the surface and dihedral scatterings are less than $45^\circ$ and larger than $45^\circ$, respectively, but their average approximates to $45^\circ$ (corresponding to the volume scattering) in DS and SD because SDoP$_s$ and SDoP$_d$ are comparable. This misleads $H/\alpha_{cp}$ to identify SD and DS as volume scattering.

VI. UNSUPERVISED CLASSIFICATION BASED ON THE STATISTICAL UNDERSTANDING OF UHD

Besides the work on theoretical unification and practical application of HTD, another aim of this paper is to investigate the potential unifiability between HTD and CD. Instead of treating them as two competitive approaches, we focus on their consistency. The consistency on dominant single target extraction has been investigated in Section IV, and this section will further explore the consistency on the description of mixed scattering. The Cloude–Pottier statistical modeling of random scatterer is extended to UHD, which enables another application of UHD to classification.

### TABLE III

<table>
<thead>
<tr>
<th>Scenario I: One dominant preference</th>
<th>Scenario II: Two dominant preferences</th>
<th>Scenario III: Random scatterer (R)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2/3 ≤ SDoP$_3$ ≤ 2/5 SDoP$_3$</td>
<td>2/5 ≤ SDoP$_3$ &lt; 2/3 SDoP$_3$</td>
<td>$SDoP_3 &lt; 2/5$</td>
</tr>
<tr>
<td>S</td>
<td>D</td>
<td>V</td>
</tr>
<tr>
<td>SD</td>
<td>DS</td>
<td>DV</td>
</tr>
<tr>
<td>SV</td>
<td>VS</td>
<td>SDV</td>
</tr>
<tr>
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<td>VS</td>
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<td>DV</td>
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<td>DSV</td>
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### TABLE IV

<table>
<thead>
<tr>
<th>Percentages of Appearance of the Ten Scattering Classes in the Scattering Preference-Based Classifications of the San Francisco Scene with and without Target Deorientation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Items</td>
</tr>
<tr>
<td>No deorientation</td>
</tr>
<tr>
<td>Deorientation</td>
</tr>
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</table>

### TABLE V

<table>
<thead>
<tr>
<th>Quantitative Evaluation of the Scattering Preference-Based Pyramid Scheme and the Entropy/Alpha Scheme on Scattering Scenario II by Surveying How Many Scenes of Z3, Z4, and Z5 also Appear in SD, SV, DS, DV, VS, and VD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Items</td>
</tr>
<tr>
<td>$H/\alpha_{cp}$</td>
</tr>
<tr>
<td>Z3</td>
</tr>
<tr>
<td>Z4</td>
</tr>
<tr>
<td>Z5</td>
</tr>
</tbody>
</table>
A. Statistical Modeling of UHD

The single target scattering $k_{Si}$ in UHD is independently extracted by nine dichotomies preferring different scatterings. UHD hence provides nine potential understandings of mixed target scattering. We normalize $k_{Si}$ into a unit vector $k_{nSi}$ and parameterize $k_{nSi}$ in accordance with the revised Bragg scattering $\alpha - \beta$ model [38]

$$k_{nSi} = \frac{k_{Si}}{||k_{Si}||_F} = e^{j\varphi_i} \begin{bmatrix} \cos \alpha_i \\ \sin \alpha_i \cos \beta_i e^{j\delta_i} \\ \sin \alpha_i \sin \beta_i e^{j\gamma_i} \end{bmatrix}, \ i = 1, 2, \ldots, 9$$

(27)

where $\varphi_i$ is an absolute phase. $k_{nSi}$ is treated as a potential form of the real scattering occurring with probability

$$P_i = \frac{\text{SPAN}_{Si}}{\sum_{i=1}^{9} \text{SPAN}_{Si}} = \frac{||k_{Si}||^2_F}{\sum_{i=1}^{9} ||k_{Si}||^2_F}, \ i = 1, 2, \ldots, 9.$$  

(28)

We call this the nine-symbol Bernoulli scattering model. Then, each parameter in (27), such as $\alpha$, can be treated as a random sequence $\{\alpha_i\}$ with occurrence probability $P_i$. Hence, their mean estimations are obtained by

$$\bar{\alpha} = \sum_{i=1}^{9} P_i \alpha_i, \ \bar{\beta} = \sum_{i=1}^{9} P_i \beta_i, \ \bar{\delta} = \sum_{i=1}^{9} P_i \delta_i, \ \bar{\gamma} = \sum_{i=1}^{9} P_i \gamma_i.$$  

(29)

By replacing the parameter with its average, a mean estimation of target scattering is achieved according to (27). The Bernoulli model also provides us another estimation of the average SDoP ($\text{SDoP}_9$)

$$\text{SDoP}_9 = \sum_{i=1}^{9} P_i \text{SDoP}_{Di} = \frac{\sum_{i=1}^{9} \text{SPAN}_{Si}^2}{\text{SPAN} \sum_{i=1}^{9} \text{SPAN}_{Si}}.$$  

(30)

$\text{SDoP}_9$ changes to $\text{SDoP}_3$ if vectors $k_{S1}$, $k_{S4}$, and $k_{S7}$ are used for estimation only. It also measures the target randomness because it is 1 for single target, is 1/3 for noisy target, and is between 1/3 and 1 for the others.
SDoP is compared with entropy $H$ in the following. Fig. 6(a) and (b) shows the obtained $H$ and SDoP on the San Francisco data. They behave completely inversely, and Fig. 6(d) exhibits their correlation further. A good correspondence is observed for the target of high $H$, but a bad correspondence arises for low- and medium-$H$ targets. $H$ is only related to eigenvalue $\lambda_j$ and is independent of eigenvector $u_j$. However, given $\lambda_j$, a series of $T$ is obtained by varying $u_j$ and so is SDoP. This is the main factor leading to the ambiguous mapping from SDoP to $H$. The black contour in Fig. 6(d) shows the maximum range of the influence of $\lambda_j$ when $u_j$ is excluded, where the outside areas are just from the additional contribution of $u_j$. Nevertheless, the influence of $u_j$ is controlled by the randomness, and it does not function only when the target is fully determined or noisy. $u_j$ is the dominant scattering in the low-randomness scenario. The fluctuation of $u_j$ then leads to a wide range of SDoP and signifies the ambiguity. The contributions of $u_1$, $u_2$, and $u_3$ are comparable in the high-randomness case, and their orthogonality then restricts the influence of $u_j$ and results in the good mapping.

Another contribution to the ambiguous mapping is from the nonlinear logarithmic operation in the calculation of $H$. To illustrate this, we define an average SDoP for CD by substituting $\lambda_j$ into (30) for SPAN,$Si,

\[
\text{SDoP}_{cp} = \frac{\sum_{j=1}^{3} \lambda_j^2}{\left(\sum_{j=1}^{3} \lambda_j\right)^2} = \frac{\|T\|_F^2}{\text{SPAN}^2}.
\]

The SDoP$cp$ of the San Francisco scene is shown in Fig. 6(c). It performs similarly as SDoP$9$, Fig. 6(e) shows the relationship between SDoP$9$ and $H$, by which the influence of nonlinearity is obviously revealed. Both $H$ and SDoP$9$ depend on $\lambda_j$ only; their correspondence is hence confined by the black contour. Because the logarithmic operation is avoided, the relationship between SDoP$9$ and SDoP$cp$ given in Fig. 6(f) presents the clearest display of the influence of target randomness on the mapping. We can conclude from these observations that, if $H$ is a global feature in measurement of target randomness, then SDoP$9$ is just a local one which reveals more details.

In view of these observations, we further use SDoP$9$ to differentiate the three scattering scenarios. The values of 2/3 and 2/5 are still used as the boundaries because they also correspond well to the boundaries of $H$, as shown in Fig. 6(d). Table VI lists the updated three scattering scenarios.$2$

The average $\alpha$ angle estimation in (29) is detailed here

\[
\alpha_\beta = \frac{9}{i=1} P_i \alpha_i \text{with } \alpha_i = \arccos \left( \frac{|k_{Si}(1)|}{\|k_{Si}\|_F} \right), \text{ } i = 1, 2, \ldots, 9.
\]

To differentiate from the Cloude–Pottier alpha angle $\alpha_{cp}$, we call (32) the Li–Zhang alpha angle ($\alpha_{\beta}$) hereafter.$3$ One can easily validate that $\alpha_z$ corresponds to a flat change from surface scattering ($\alpha_z = 0^\circ$) to dihedral scattering ($\alpha_z = 90^\circ$). It thus performs similarly as $\alpha_{cp}$ to characterize the target scattering mechanism. Fig. 7(a) and (d) illustrates these two angles on the San Francisco scene. They behave nearly the same, and their strong correlation in Fig. 7(b) clearly shows this, where the black curve gives the contribution of $\lambda_j$ when $u_j$ is excluded. Once again, the correspondence is impacted by $u_j$ and controlled by randomness. $\alpha_z$ is equal to $\alpha_{cp}$ only when the target is completely determined, and good mapping is kept until low-randomness scenario I, as exhibited in Fig. 7(c), which changes, however, in scenarios II and III, as illustrated in Fig. 7(e) and (f). Nevertheless, a good correspondence is still observed. Table VII lists the mean and standard deviation of the absolute residue between the two angles (i.e., $|\alpha_z - \alpha_{cp}|$) on each of the three scenarios and on the total scene to quantify the influence of randomness. The total mean and standard deviation are only 2.44$^\circ$ and 1.91$^\circ$, respectively.

### B. SDoP/Alpha Classification Scheme

The consistency between $\alpha_z$ and $\alpha_{cp}$ enables us to similarly use $\alpha_z$ to subdivide the three SDoP$9$-scenarios into eight zones. To validate this, we first transfer the 2-D $H/\alpha_{cp}$ plane of San Francisco in Fig. 8(a) into the SDoP$9 - \alpha_z$ space, as shown in Fig. 8(b). The zones are still clearly separable with slight mixtures near borders. Hence, the combination of SDoP$9$ and $\alpha_z$ is of good separability. By referring to the $\alpha_{cp}$ boundaries in $H/\alpha_{cp}$ and the $\alpha_z - \alpha_{cp}$ correlation in Fig. 7, the $\alpha_z$ partition of each scenario is obtained and listed in Table VI. The corresponding 2-D SDoP$9/\alpha_z$ plane of San Francisco is displayed in Fig. 8(c). By further mapping the classification to $H - \alpha_{cp}$ space, we get Fig. 8(d), where the horizontal borders are straight due to the good $\alpha_z - \alpha_{cp}$ correspondence, but the vertical borders are somewhat askew, particularly the one between Z5 and Z8, because the SDoP$9 - H$ correspondence is not so good. Nevertheless, the majority of Fig. 8(d) is correctly aligned to the $H/\alpha_{cp}$ plane in Fig. 8(a). Figs. 5(e) and 9(a) exhibit the two classification results. The typical scatterers, like ocean, city, and vegetation, are clearly separated with nice consistency achieved.

---

2Although the attention here is paid on SDoP$9$, SDoP$cp$ also deserves our notice. Both the scattering diversity parameter developed by Praks et al. [47] and the $h$ parameter proposed by An et al. [48] which behave as two alternates to entropy $H$, in fact, directly relate to it. The right side of (31) gives a fast derivation of SDoP$cp$, independent of eigendecomposition. Thus, it is also a fast and competent alternative to $H$.

3The authors would like to thank the reviewer for the suggestion of naming this angle after their names.
Fig. 7. Comparison of (a) the Cloude–Pottier alpha angle ($\alpha_{cp}$) and (d) the Li–Zhang alpha angle ($\alpha_{lz}$) on the scene of San Francisco, where (b), (c), (e), and (f) display the $\alpha_{cp} - \alpha_{lz}$ correlation on the total scene and on the three scattering scenarios, respectively. The black curves show the corresponding relationships on the diagonal coherence matrix.

### TABLE VII

**Statistical Characters of the Absolute Residue Between the Cloude–Pottier Alpha Angle and the Li–Zhang Alpha Angle on the Three Scattering Scenarios and on the Total Scene of San Francisco**

<table>
<thead>
<tr>
<th>Items</th>
<th>Scenario I</th>
<th>Scenario II</th>
<th>Scenario III</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>$1.34^\circ$</td>
<td>$3.36^\circ$</td>
<td>$2.18^\circ$</td>
<td>$2.44^\circ$</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>$1.04^\circ$</td>
<td>$2.06^\circ$</td>
<td>$1.43^\circ$</td>
<td>$1.91^\circ$</td>
</tr>
</tbody>
</table>

The confusion matrix [48], a statistical table to record the consistency of two classifications, is used here to quantitatively evaluate the two classifications. In Table VIII, each row reflects how many pixels of $Z_i$ of SDoP$_9/\alpha_{lz}$ are reclassified into different zones of $H/\alpha_{cp}$. The majority of $Z_i$ in SDoP$_9/\alpha_{lz}$ has been correctly classified into the corresponding $Z_i$ in $H/\alpha_{cp}$. Table IX shows their total consistency of 94.04%. The main visual difference arises in the mountain area of rectangle 8, where some volume scatterings (green) in Fig. 5(e) change into rough surface scatterings (yellow) in Fig. 9(a), but it is not easy to identify the better one from them. By referring to the classification in Fig. 5(d), the changing area belongs to SD or SV denoting more preferable to surface scattering. Thus, SDoP$_9/\alpha_{lz}$ performs better. To eliminate the potential biases from the data set, the two classifications are further compared on the scenes of Oberpfaffenhofen (DLR L-band ESAR) and Flevoland (NASA/JPL L-band AIRSAR). Table IX lists their consistencies, still as high as 91.64% and 91.29%. SDoP$_9/\alpha_{lz}$ is thus a competent alternative to $H/\alpha_{cp}$.

#### C. Influence of Target Orientation

SDoP$_9$ and $\alpha_{lz}$ are not roll-invariant; thus, a better classification may be obtained if deorientation is performed first. However, the classification of deoriented San Francisco in Fig. 9(b) does not make much difference from Fig. 9(a). To show this, Table X counts how many pixels of the scene are attributed into $Z_i$ by $H/\alpha_{cp}$ as well as SDoP$_9/\alpha_{lz}$ with and without deorientation. Both of the two SDoP$_9/\alpha_{lz}$ results are consistent with $H/\alpha_{cp}$, but the changes after deorientation are not as significant as those in Table IV. These reveal that, even if SDoP$_9$ and $\alpha_{lz}$ are not roll-invariant, however, a certain invariance is achieved when they are averagely integrated into
This article has been accepted for inclusion in a future issue of this journal. Content is final as presented, with the exception of pagination.

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Fig. 8. Consistency comparison of (a) entropy/alpha ($H/\alpha_{cp}$) classification plane and (c) SDoP/alpha ($\text{SDoP}_9/\alpha_{lz}$) classification plane on scene of San Francisco. (b) Equivalent presentation of (a) in SDoP$_9\alpha_{lz}$ space. (d) Equivalent presentation of (c) in $H - \alpha_{cp}$ space.

SDoP$_9$ and $\alpha_{lz}$. Deorientation then has limited improvement and is not always necessary. This is also given in Table IV. The changes after deorientation are remarkable on the ten classes, which are identified by directly comparing SDoP$_s$, SDoP$_d$, and SDoP$_v$. However, the changes are only 1.97%, 2.20%, and 0.23% on the three scenarios because they are determined by SDoP$_3$, which is the statistical average of SDoP$_s$, SDoP$_d$, and SDoP$_v$. A comparison of SDoP$_3$ and SDoP$_9$ is also achieved if we further count the changes on the three SDoP$_9$-scenarios. They are −0.15%, 0.11%, and 0.04%, respectively, gentler than the former case. SDoP$_9$ is thus more stable because it involves the average of all of the nine single target scatterings.

VII. DISCUSSION

Sections V and VI provide two descriptions for mixed target scattering in terms of the adaptive permutation and statistical modeling of UHD, respectively. They enable not only a novel description of target randomness (SDoP$_3$ or SDoP$_9$) but also two different yet consistent indications of scattering mechanism ($\alpha_{lz}$ and the permutation of SDoP$_{D1}$). The scattering preference plays a pivotal role in both approaches.

Besides the influence of target orientation, another factor that we should note when using the two descriptions is the speckle filtering. The proposed classifications lay their basis on parameters SDoP$_{D1}$, SDoP$_s$, SDoP$_d$, and $\alpha_{lz}$, which are retrieved by UHD from $T$. Incoherent averaging of a large number of neighboring pixels of $T$ is thus required because

Fig. 9. SDoP/alpha classifications of San Francisco data (a) without and (b) with target deorientation. The coded color of each zone is given in Fig. 8.
insufficient averaging will produce biased retrieval and hence biased classification. Lopez-Martinez et al. [49] and Lee et al. [50] have investigated the effects of speckle filtering on $H/\alpha_{cp}$, and they suggested using $7 \times 7$ and $5 \times 5$ or larger averaging window, respectively, for the estimation of $H$ and $\alpha_{cp}$. Lee and Pottier [3] further found that the $H$ estimation from RLF$_7$ performs even better than that from boxcar filter, although RLF$_7$ may not provide enough averages for reliable estimates.

This section gives a simple evaluation of the influences of speckle filtering on RLF$_n$, on the scattering pyramid and SDoP$_{3/\alpha_z}$. This can be understood by simply filtering the data with RLF$_7$, RLF$_5$, RLF$_3$, or RLF$_9$, which makes the yellow- and green-coded areas increased and improves the identifiability for beaches, park, and avenues. Such improvement is mainly ascribed to the increase of scattering randomness. It is observed that the $H$ value increases while the values of SDoP$_3$ and SDoP$_9$ decrease with the amount of averaging. However, besides introducing the square imprints, the influence of filtering on SDoP$_{Di}$, $\alpha_{lz}$, and $\alpha_{cp}$ is limited. A similar finding on $H$ and $\alpha_{cp}$ has been presented in [49] and [50]. Hence, although SDoP$_3$ and SDoP$_9$ are more independent on target orientation, SDoP$_{Di}$ is less dependent on speckle filtering. From RLF$_9$ to RLF$_7$, and further to RLF$_7$, the dark green-coded class R of scattering pyramid classification, as well as the white-coded Z1 and dark green-coded Z2 of SDoP$_{3/\alpha_z}$ and $H/\alpha_{cp}$ classifications, becomes much clearer, which reflect the increase of randomness. However, their changes from RLF$_7$ to RLF$_9$ are relatively weak. A larger window, of course, provides better estimation of $H$, SDoP$_3$, and SDoP$_9$ but at the risk of degrading the spatial resolution. If we take the compromise between resolution and parameter estimation into consideration, RLF$_7$ is sufficient for the three classifications.

The black line framed patch 9 in Fig. 10(d) denotes an urban area of San Francisco named South of Market (SoMa). Fig. 11 displays a close-up view of the class R in the scattering pyramid classifications of SoMa under RLF$_n$. Class R denotes the

### TABLE VIII

<table>
<thead>
<tr>
<th>Items</th>
<th>Z1</th>
<th>Z2</th>
<th>Z3</th>
<th>Z4</th>
<th>Z5</th>
<th>Z6</th>
<th>Z7</th>
<th>Z8</th>
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<tbody>
<tr>
<td>$H/\alpha_{cp}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td>Z1</td>
<td>87.54%</td>
<td>9.71%</td>
<td>2.75%</td>
<td>0</td>
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<td>0</td>
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<td>Z2</td>
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<td>0.14%</td>
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<td>5.60%</td>
<td>89.27%</td>
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<td>0.01%</td>
<td>0.01%</td>
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<td>0</td>
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<td>0</td>
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<td>0.34%</td>
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<td>0</td>
<td>0</td>
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<td>0</td>
<td>3.51%</td>
<td>77.56%</td>
<td>3.51%</td>
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<td>Z8</td>
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<td>0</td>
<td>0.04%</td>
<td>0.62%</td>
<td>0</td>
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<td>99.32%</td>
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### TABLE IX

<table>
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<th>Flevoland</th>
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<tr>
<td>No deorientation</td>
<td>94.04%</td>
<td>91.64%</td>
<td>91.29%</td>
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<tr>
<td>Deorientation</td>
<td>94.21%</td>
<td>91.45%</td>
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### TABLE X

<table>
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<th>Items</th>
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<th>Z3</th>
<th>Z4</th>
<th>Z5</th>
<th>Z6</th>
<th>Z7</th>
<th>Z8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H/\alpha_{cp}$</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SDoP$_{3/\alpha_z}$ without deorientation</td>
<td>2.15%</td>
<td>9.78%</td>
<td>27.27%</td>
<td>14.78%</td>
<td>6.76%</td>
<td>2.88%</td>
<td>0.15%</td>
<td>36.23%</td>
</tr>
<tr>
<td>SDoP$_{3/\alpha_z}$ with deorientation</td>
<td>1.95%</td>
<td>9.80%</td>
<td>26.98%</td>
<td>13.73%</td>
<td>7.78%</td>
<td>3.23%</td>
<td>0.17%</td>
<td>36.36%</td>
</tr>
</tbody>
</table>

course are clear in Fig. 10(a), but they are hard to be identified from Fig. 10(e) and (i). The three classifications are greatly strengthened by simply filtering the data with RLF$_3$, RLF$_5$, RLF$_7$, or RLF$_9$, which makes the yellow- and green-coded areas increased and improves the identifiability for beaches, park, and avenues. Such improvement is mainly ascribed to the increase of scattering randomness. It is observed that the $H$ value increases while the values of SDoP$_3$ and SDoP$_9$ decrease with the amount of averaging. However, besides introducing the square imprints, the influence of filtering on SDoP$_{Di}$, $\alpha_{lz}$, and $\alpha_{cp}$ is limited. A similar finding on $H$ and $\alpha_{cp}$ has been presented in [49] and [50]. Hence, although SDoP$_3$ and SDoP$_9$ are more independent on target orientation, SDoP$_{Di}$ is less dependent on speckle filtering. From RLF$_9$ to RLF$_7$, and further to RLF$_7$, the dark green-coded class R of scattering pyramid classification, as well as the white-coded Z1 and dark green-coded Z2 of SDoP$_{3/\alpha_z}$ and $H/\alpha_{cp}$ classifications, becomes much clearer, which reflect the increase of randomness. However, their changes from RLF$_7$ to RLF$_9$ are relatively weak. A larger window, of course, provides better estimation of $H$, SDoP$_3$, and SDoP$_9$ but at the risk of degrading the spatial resolution. If we take the compromise between resolution and parameter estimation into consideration, RLF$_7$ is sufficient for the three classifications.

The black line framed patch 9 in Fig. 10(d) denotes an urban area of San Francisco named South of Market (SoMa). Fig. 11 displays a close-up view of the class R in the scattering pyramid classifications of SoMa under RLF$_n$. Class R denotes the
Fig. 10. Evaluation of the influences of refined Lee filtering with (first column) $1 \times 1$ (i.e., without filtering), (second column) $3 \times 3$, (third column) $5 \times 5$, and (fourth column) $9 \times 9$ large aligned windows on the classifications of (first row) the scattering pyramid, (second row) entropy/alpha, and (third row) SDoP/alpha. Patch 9 in (d) denotes an urban area of San Francisco named South of Market (SoMa), the classification of which is detailed in Fig. 11 to evaluate the performance of the scattering pyramid on the discrimination of buildings. The coded color of each class of the three classifications can be found in Figs. 5 and 8.

high-randomness scenario and is split into $Z_1$ and $Z_2$ in SDoP$_9/\alpha_{lz}$ and $H/\alpha_{cp}$. It expands as the increase of filtering size and achieves stability in RLF$_7$ and then dominates patch 9 (covers about 52% of the area). Such classification may lead us to misidentify SoMa as a vegetated area because vegetation usually possesses high randomness and volume scattering. One can easily attribute this misidentification to the orientation of the building because it will increase the HV scattering and mistake building for forest. Hence, a powerful technique to cure this is just to perform the deorientation operation to minimize the HV scattering. However, deorientation is found no longer effective here because all of the classifications in Figs. 10 and 11 have been grounded on the deoriented data.

SoMa is now becoming a new test for target decomposition procedures because of its special city planning. Different from other areas of San Francisco, the streets here are neither vertical nor horizontal but are about 40° tilted. We can roughly observe this from the classification in Fig. 11(d). Based on the truth that a building is usually constructed along the streets, such city planning will result in a misalignment (azimuth tilting) between the vertical wall of the building and the azimuth direction of the radar. The wall normal will not be within the radar incidence plane then, and an orientation is thus created. The detailed analysis and modeling of the polarization orientation in such built-up area has been presented by Kimura [51]. Moreover, azimuth tilting in a dense urban area will also increase the scattering complexity and randomness, and the wide distribution of class R has reflected the high randomness in SoMa. Patch 10 in Fig. 10(d) is also an urban area adjacent to SoMa with dense buildings. Nevertheless, the wide distribution of dark red color (class DS) indicates its medium randomness because of the negligible azimuth tilting here. Instead of target orientation, it is the high randomness ($H > 0.9$, and SDoP$_3$ and SDoP$_9 < 2/5$) that enables the scattering pyramid, SDoP$_9/\alpha_{lz}$, and $H/\alpha_{cp}$ to attribute SoMa to class R, as well as to $Z_1$ and $Z_2$.

Azimuth tilting results in target orientation and high randomness and increases the possibility for misidentifying buildings as vegetation. Deorientation can only compensate the orientation-related misidentification but has tiny effects on that from randomness, as shown in Section VI-C, parameters SDoP$_3$, SDoP$_9$, and $H$ possess a strong invariance to $U_\varphi$. As for such randomness-related misidentification, from the viewpoint of model-based target decompositions, we may need to develop modeling that better accounts for the random scattering. The superiority of Singh’s decomposition over Yamaguchi’s and Sato’s decompositions in SoMa may be ascribed to this reason [52]. Class R in the scattering pyramid
scheme denotes a chaotic state where the three preferences $\text{SDoP}_s$, $\text{SDoP}_d$, and $\text{SDoP}_v$ are comparable. It is hence determined only by $\text{SDoP}_3$ with no comparison of the three preferences. Nevertheless, there are still tiny differences among $\text{SDoP}_s$, $\text{SDoP}_d$, and $\text{SDoP}_v$, and this information may help us discriminate buildings from forest. By treating $\text{SDoP}_d$, $\text{SDoP}_v$, and $\text{SDoP}_s$, respectively, as the colors of red, green, and blue, the second row in Fig. 11 shows that the pseudocolor map of the pixels of SoMa belonged to class R under RLF. It can be seen that the saturation of such characterization is relatively low because of the comparable preferences here. Nevertheless, one can still discriminate the hue difference and find more red colors than green colors. The average of $\text{SDoP}_d$ in Fig. 11(i) is 0.3618, larger than that of $\text{SDoP}_v$ (0.3094), which indicates the preference of buildings to vegetation. Therefore, the scattering preference analysis can be further used to elaborate the classifications of scattering pyramid and $\text{SDoP}_9/\omega_{\ell 2}$. They together provide a coarse-to-fine understanding of different scattering behaviors.

VIII. CONCLUSION

This paper has involved three aspects to reply to the existing concerns on HD. The first aspect concerns the nonuniqueness, which is treated by generalizing HD to UHD. UHD covers all of the existing HTD and provides a fair application mechanism for each HTD. The second aspect regards the practical value of HTD. Scattering preference is identified as a special characteristic of HTD and enables us to relate each dichotomy directly to a canonical scattering. The information is adaptively used in the description of mixed scattering based on a pyramid scheme and promotes a scattering preference-based classification, which is demonstrated to have better discrimination of targets than entropy/alpha. This overturns the general impression that HTD is inferior to CD. We thus conclude that HTD is born with physical significance and nice applicability. We hope that this will promote the wide acceptance of such decompositions in the future. The third aspect regards the uniqueness controversy between HTD and CD. Instead of treating them as two competitive methods, their potential unification is investigated on two aspects, i.e., optimal extraction of single target and statistical description of mixed scattering. We show that UHD and CD can achieve consistent target extraction, and the proposed $\text{SDoP}$/alpha classification is a competent alternative to entropy/alpha. These give a good ending to the controversy between Huynen and Cloude.

The four decades’ development of this area has indicated that there is no unique decomposition but rather an infinity. Uniqueness arises only when one prefers a certain aspect. Each decomposition has its own usefulness but cannot provide all of the information regarding target scattering. Therefore, we need to combine all of the decompositions for an integrated understanding of target decompositions to make the polarimetric radar viewing of the world more colorful.

APPENDIX

THEORETICAL PROOF OF THE CONSISTENCY AND DIFFERENCE BETWEEN CHD AND YD

Yang et al. [14] proposed a modified HD (i.e., YD) in terms of two transforms of $K$, which can be concisely expressed via
matrix $T$ as
\[
    T_1 = U_1^H T_1 U_1^H = \begin{bmatrix}
    T_{22} & T_{21} & -jT_{23} \\
    T_{12} & T_{11} & -jT_{13} \\
    jT_{32} & jT_{31} & T_{33}
    \end{bmatrix},
    \]
\[
    T_2 = U_2^H T_2 U_2^H = \begin{bmatrix}
    T_{33} & -T_{31} & jT_{32} \\
    -T_{13} & T_{11} & -jT_{12} \\
    -jT_{23} & jT_{21} & T_{22}
    \end{bmatrix}.
\] (A1)

They, respectively, correspond to the matrices $K_1$ and $K_2$ mentioned in Section II-B, where
\[
    U_1 = \begin{bmatrix}
    0 & 1 & 0 \\
    1 & 0 & 0 \\
    0 & 0 & j
    \end{bmatrix}, \quad U_2 = \begin{bmatrix}
    0 & 0 & -1 \\
    1 & 0 & 0 \\
    0 & j & 0
    \end{bmatrix}.
\] (A2)

Based on these, YD performs as follows [14].

If $T_{11}$ is larger than $T_{22}$ and $T_{33}$, then directly apply HD to $T$, and denote
\[
    T = T_S + T_N. \tag{A3}
\]

If $T_{22}$ is larger than $T_{11}$ and $T_{33}$, then turn to apply HD to $T_1$, and denote
\[
    T_1 = T_{S1} + T_{N1}. \tag{A4}
\]

and the modified decomposition of $T$ is
\[
    T = U_1^H T_1 U_1 = U_1^H T_{S1} U_1 + U_1^H T_{N1} U_1 = T_{MS1} + T_{MN1}. \tag{A5}
\]

While if $T_{33}$ is larger than $T_{11}$ and $T_{22}$, then apply HD to $T_2$, and denote
\[
    T_2 = T_{S2} + T_{N2}. \tag{A6}
\]

and the modified decomposition of $T$ is
\[
    T = U_2^H T_2 U_2 = U_2^H T_{S2} U_2 + U_2^H T_{N2} U_2 = T_{MS2} + T_{MN2}. \tag{A7}
\]

From the vector $k_{S1}$ representation of HD in (14), we obtain that
\[
    T_{S1} = k_{11} k_1^H = \frac{1}{T_{22}} \begin{bmatrix}
    T_{22} & T_{12} & -jT_{23} \\
    T_{12} & T_{11} & -jT_{13} \\
    jT_{32} & jT_{31} & T_{33}
    \end{bmatrix} \begin{bmatrix}
    T_{22}^* & T_{12}^* & -jT_{23}^*
    \end{bmatrix}.
\] (A8)

Therefore, the modified single targets $T_{MS1}$ and $T_{MS2}$ in (A5) and (A7) can be further expressed as
\[
    T_{MS1} = U_1^H T_{S1} U_1 = \frac{1}{T_{22}} \begin{bmatrix}
    |T_{12}|^2 & T_{12}^* T_{22} & T_{12} T_{22}^*
    \end{bmatrix} \begin{bmatrix}
    T_{12}^* T_{22} & |T_{22}|^2 & T_{22}^* T_{12}\n    T_{12} T_{22}^* & T_{22}^* T_{12} & |T_{12}|^2
    \end{bmatrix}.
\] (A10)

\[
    T_{MS2} = U_2^H T_{S2} U_2 = \frac{1}{T_{33}} \begin{bmatrix}
    |T_{13}|^2 & T_{13}^* T_{23} & T_{13} T_{23}^*
    \end{bmatrix} \begin{bmatrix}
    T_{13}^* T_{23} & |T_{23}|^2 & T_{23}^* T_{13}\n    T_{13} T_{23}^* & T_{23}^* T_{13} & |T_{13}|^2
    \end{bmatrix}.
\] (A11)

They correspond to the single target Pauli vectors $k_{S4}$ and $k_{S7}$ in (17) and (18), respectively:
\[
    T_{MS1}^{k_{S4}} = \frac{1}{\sqrt{B_0 + B}} \begin{bmatrix}
    C - jD \\
    B_0 + B \\
    E - jF
    \end{bmatrix} = \frac{1}{T_{22}} \begin{bmatrix}
    T_{12} \\
    T_{22} \\
    T_{32}
    \end{bmatrix}. \tag{A12}
\]

\[
    T_{MS2}^{k_{S7}} = \frac{1}{\sqrt{B_0 - B_0}} \begin{bmatrix}
    H + jG \\
    E + jF \\
    B_0 - B
    \end{bmatrix} = \frac{1}{T_{33}} \begin{bmatrix}
    T_{13} \\
    T_{23} \\
    T_{33}
    \end{bmatrix}. \tag{A13}
\]

Therefore, the three target dichotomies (A3), (A5), and (A7) in YD are the same as those in CHD. YD selects them based on the $2A_0$ parameters of $T$, $T_1$, and $T_2$, i.e., $T_{11}$, $T_{22}$, and $T_{33}$, but CHD achieves this by comparing the SPAN of $T_S$, $T_{MS1}$, and $T_{MS2}$.

**ACKNOWLEDGMENT**

The authors would like to thank the National Aeronautics and Space Administration/Jet Propulsion Laboratory and the German Aerospace Center for providing the related PolSAR data. The authors would like to thank the anonymous reviewers for their valuable comments and suggestions. The authors would also like to thank the associate editor for the editorial correction of this paper.

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