Spectral fluctuation analysis of ionospheric inhomogeneities over Brazilian territory. Part I: Equatorial F-region plasma bubbles

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Abstract

In this Part I of a more general paper on the analysis of ionospheric irregularities over Brazilian territory, we apply the Detrended Fluctuation Analysis (DFA) method to evaluate in situ equatorial F-region plasma bubbles events carried out with a sounding rocket over equatorial region in Brazil. The range of scaling exponents derived from the DFA technique are compared to previous results obtained using Power Spectral Density (PSD) technique (which is widely used in this area despite its recognized inaccuracy to analyze short series). The results obtained in this first part of our investigation, using DFA, also show a wide range of spectral index variation with standard deviation of the same order found from the previous application using PSD (σm ≃ 10%). Therefore, since the dependence of the technique are disregarded, our findings also supports that the observed lack of a universality class characterized by the nonexistence of a single spectral index (with σm ≃ 2%) may be due to non-homogeneity energy cascades that can appear in the incoherent ionospheric turbulent process.

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1. Introduction

Ionospheric irregularities have been measured across different equatorial sectors (e.g. McClure et al., 1977; Kelley et al., 1982; Sinha and Raizada, 2000; Muralikrishna, 2006; Wiens et al., 2006; Kelley, 2009; Chapagain et al., 2011), so that the interpretation of several complex ionospheric plasma processes has progressed considerably, both in theory and observations. In selected previous work (see Table 1) the authors seek to characterize using the Power Spectral Density (PSD) calculated from electric field fluctuations) the presence of a typical turbulence process which can be explained from the plasma instabilities provided by models (Kelley, 2009). In this search scope that has spanned for more than thirty years, most of the results have shown that the ionospheric turbulence can not be explained from a single universality class as the turbulence K41 (β ≃ 1.66 with σm ≃ 2%) (Frisch, 1995). The first possible cause, before raising the complexity of alternative physical process, is the inherent bias from the technique PSD due to sample size limitations (in the area of statistical physics and even in applications on solar radio astronomy, this problem is already known (Veronese et al., 2011). Then, the main objective of this work is to verify, using Detrended Fluctuation Analysis
(DFA) (Peng et al., 1994; Veronese et al., 2011), the nature of the wide power spectral range variation ($\sigma_m > 10\%$) reported by previous equivalent analysis using PSD.

The DFA approach is complementary to Power Spectral Density (PSD) (Heneghan and McDarbyee, 2000) and establishes a direct relationship with the fractal model for turbulence (Frisch, 1995). Also DFA brings features of improved robustness for short series (the order of the amount of records $\ll 10^4$) and therefore can also be used to monitor the formation of extreme events in the spatial–temporal domain (Veronese et al., 2011). Accordingly, it is a more appropriate tool for studies of nonhomogeneous processes in ionized fluids where scaling instabilities play a key role in the fluctuation coming from the underlying physical process (Korolev and Skvorbova, 2006).

Here, as Part I of a more general approach, we present a complementary DFA spectral analysis of ionospheric F-layer plasma bubbles regions during the downleg phase where the electric field fluctuations are measured. In a next paper, Part II of this analysis, in situ measurements of ionospheric depletions found below the base of the F-region, the so-called E-Valley irregularities, will also be addressed in the same way. We hope that with the results of Part II a deeper analysis of the physical nature of $\beta$ variability and its connection with the ionospheric homogeneity as a whole, can then be somewhat better understood.

This Part I is organized as follows. In Section 2, the Equatorial F-region data set is explained. A quick explanation on DFA is presented in Section 3, while its technical aspects are presented in the Appendix A. The results and interpretations are also given in Section 3. In the last section, we present the conclusions and introduce the next steps to the Part II of this paper.

### 2. Ionospheric plasma measurements

Simultaneous in situ measurements of electric field and electron density were recorded in the course of a rocket experiment launched from the equatorial station Alcântara (2.24°S; 44.24°W), Brazil, on December 18, 1995 at 21:17 LT, under low solar flux ($F_{10.7} = 67$) and geomagnetic quiet conditions ($\Sigma K_p = 6$; $D_S T \min = -14$ nT). During a 11 min flight, where the rocket covered a horizontal range of about 589 km with an apogee of 557 km, it was detected an undisturbed electron density profile along the upleg trajectory showing the presence of a well-defined base for the F-region around $h = 300$ km, while a certain number of medium and large scale plasma bubbles were revealed during the downleg flight (Fig. 1). We selected four data samples (a, b, c and d), two very close to bubbles peaks (c and d) and two slightly off peak (a and b), but are also considered within the bubble regime (see Fig. 2, left side). Other time series sub-sample were not selected by issues of poor data quality in high resolution (number of sequential measurements of sub-sample, for technical reasons, was taken always containing at least 1024 points).

A Langmuir probe (LP) was applied to measure the fluctuations in the electron density and the electron saturation current ($I_e$), where the latter is used to determine the vertical profiles displayed in Fig. 1. The electron density $n_e$ is proportional to $I_e$ measured during 1 s in each cycle, when a fixed potential (+2.5 V) is applied to the LP sensor. The fluctuating (AC) component of the ionospheric electric field was detected by spherical sensors of an electric field double probe (EFDP) installed at the extremities of two telescopic booms, whose maximum and minimum lengths were 156 cm and 50 cm, respectively. The varying part of the potential difference between the EFDP sensors is directly

![Fig. 1. Downleg vertical profile of electron density. The gray arrows in the panel indicate the presence of plasma bubbles of different scale sizes distributed along the profile. The lower-case letters in black (a to d) indicate the regions where electric field fluctuation data were extracted for the present DFA analysis.](Image)
Fig. 2. The left column (a.1, b.1, c.1, d.1) are time series ($\delta E$) of electric field fluctuation of EFP probe during the downleg flight. Note that a, b, c and d correspond to the regions identified in Fig. 1. The right column (a.2, b.2, c.2, d.2) shows the DFA and its $\alpha$ exponent for each correspondent time series on the left.
related to the ambient electric field (EF). The EF fluctuation time series ($\delta E$) are extracted from EFDP measurements by a high-pass filter (cutoff frequency: 10 Hz), and then sampled to a rate of 0.8 ms. Due to the variation of the rocket speed along the flight, the scale sizes $\lambda$ estimated from these data will vary according to the expression $\lambda = v/f$, where $f$ is the range of observable fluctuation frequency limited by the Nyquist limit. A detailed discussion regarding the scientific payload and data processing can be found in Muralikrishna et al. (2003) and Muralikrishna (2006).

3. Fluctuation analysis, results and interpretation

Although Power Spectral Density (PSD) is commonly used to obtain a structural signature of both the scaling and nonlinearity in turbulent physical systems, it does not reveal any statistical property of nonhomogeneous energy cascades in the inertial regime of the underlying process. However, the lack of a single spectral index expected from the same stochastic dynamics (or universality statistical class signature) may indicate that the energy cascade process is not a homogeneous one. In such case, the most appropriate model for turbulence is that related to processes within the self-similarity paradigm of the fractal dynamic approach (Frisch, 1995). Within this approach it is important to analyze the temporal fluctuations whereas temporal processes may be related to the spatial variation of a structure function which has a coupling self-similarity. DFA technique allows this operation because it relates the scaling richness of a nonlinear process (such as turbulence) to the sum of the low and high frequency fluctuations. This coupling can be analyzed detrending the time series of the lowest frequencies to refine the multiscale coupling (Peng et al., 1994; Veronese et al., 2011).

The DFA operator generates a scaling function $F(x)$ from various values of $x$ scale exponent. Its spectral slope specifies a power law related with the cumulative sum of the fractal gaussian noise of the structure function as $x = H + 1$, where $H$ is the Hurst exponent. In this mathematical framework it is possible to show that DFA spectra measures scaling exponents from non-stationary self-affine fluctuations profiles being useful for characterizing fluctuation patterns that appear to be due to long-range spatial and/or temporal correlations. DFA has been used in several analysis from physiological data to signals in econometry and physics (Vandewalle and Ausloos, 1998; Heneghan and McDarbyee, 2000; Kantelhardt et al., 2002).

The detailed scheme of the DFA operator is presented in the Appendix A. It should be noted that each time series of the ionospheric electric field fluctuations ($\delta E$) (Fig. 2, left side) described in the previous section is analyzed here by DFA resulting in their respective spectra $F(x)$ (Fig. 2, right side).

Considering the relationship between PSD and DFA, $\beta \approx 2x - 1$, the values obtained by DFA analysis are compared with other works in which the spectral index, $\beta$, were obtained from the PSD analysis (see Table 1). We characterized the range of variation using the standard deviation ($\sigma_m$) between the two marginal values in the $\beta$-range, as is usual in experimental studies of turbulence where a universality class is considered only when $\sigma_m \leq 2\%$ (Frisch, 1995).

Based on the results shown in Table 1, it is possible to highlight at least two important findings:

- The wide beta variation ($\sigma_m \gg 10\%$) was also observed from the DFA analysis ($\sigma_m = 19\%$). Therefore, this result suggests that the previous published results found for $\sigma_m$ are not due to a possible bias from the PSD analysis.
- The values for $\langle \beta \rangle$ and $\sigma_m$, including this work, show that the electric field fluctuations, that are associated with irregularities called F-region plasma bubbles, do not correspond to the turbulent universality class with homogeneous energy cascade as from K41. Furthermore, they point to the nonexistence of a specific spectral index value (or scaling exponent), which could justify the hypothesis of a single turbulent process into the whole ionosphere. That is, considering a representing mean value from all $\langle \beta \rangle$, we get $\beta = 2.66 \pm 0.63$. Note that, the value of $\sigma_m = 63\%$ drops any class of statistical universality for the ionospheric turbulence.

Considering the two previously highlighted aspects, a direct consequence is that the DFA values admit an approach through a model of nonhomogeneous turbulence typically described through the fluctuations that can be generated from fractal Gaussian noise coupling. The theoretical approach of fractal model of turbulence assumes that a strong coupling exists between self-similarity and temporal correlation improving the efficiency of turbulent diffusion mechanism without a rigid universality class as from K41 homogeneous theory from where $\beta = -1.66$ with $\sigma_m = 2\%$ (Frisch, 1995). It is important to mention that a spectral index value outside the accuracy range ($\sigma_m < 2\%$) can also indicate the presence of the so-called higher-order spectra such as the bispectrum or bicoherence e.g. (e.g. Farge, 2005). Recent studies of the ionospheric density fluctuations in the higher altitudes suggest that the PSD fluctuations can be corrected by bispectra (Spicher et al., 2013). However, considering the scope of this article (which uses DFA) we will not step in addressing this type of treatment, which may be considered an alternative for future work.

4. Concluding remarks

In summary, the results found from ionospheric DFA analysis are useful to verify the nature of the wide power spectral range variation reported in previous articles ($\sigma_m \gg 10\%$). The first possible cause was the inherent bias from the technique PSD due to sample size limitations. In this sense, our results show that a more robust technique (such as DFA) also features the same wide variation ($\sigma_m \gg 19\%$), thus ruling out the hypothesis of technique
dependence, a subject still little discussed by researchers applying PSD on ionospheric data. Since the dependence of the technique are disregarded, our findings also supports that the observed irregularities may be due to the nature of homogeneous turbulence that grows from certain types of plasma instability. In this sense it is extremely important to analyze data from the region known as E-Valley, since it is known that plasma instabilities present in this region are different from what the theory predicts for the F-region irregularities analyzed in this Part 1 (Kelley, 2009). Then, to address the inhomogeneous turbulence as the possible cause of the observed wide spectral variation, we would need more data which will be added soon to the continuity of the proposed research (Part 2 of this paper). The subsequent article will be on improving the analysis of a larger set of data which will enable us to address the possible types of physical instabilities that lead to observed spectra and their typical both multiscaling and multifractal models which are beyond the existence of a single universality class as it should be for homogeneous ionospheric turbulence.

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Appendix A. Detrended Fluctuation Analysis

Detrended Fluctuation Analysis (DFA) can be described as a method that analyses the long-range correlations of series avoiding spurious detection of apparent correlations. Since its first appearance, in 1994, as a new approach to analyse DNA nucleotides (Peng et al., 1994), the method had quickly spread into many other fields. Can be mentioned meteorology, geology, hydrology, biology, economics and medicine (Chen et al., 2005; Ivanova and Ausloos, 1999; Del Pin et al., 2008; Montanari et al., 2000; Bahar et al., 2007; Vandewalle and Ausloos, 1998; Kantelhardt et al., 2002).

The DFA procedure can be described in three steps. Assuming x as a finite one-dimensional series with a length \( N \), \( \bar{x} \) the mean of the series and \( s \) the size of the box, the method is defined as follows.

Firstly, subtract each cumulative sum of the series by its mean. Then, integrate to obtain the “profile” \( y(i) \). In other words,

\[
y(i) = \frac{1}{N} \sum_{k=1}^{N} (x_k - \bar{x})
\]

where \( i = 1, 2, \ldots, N \).

After obtaining the profile \( y(i) \), divide it by \( N_s = \text{int}(N/s) \) non-overlapping segments with length \( s \). There may be cases where the scale \( s \) does not divide perfectly the profile, remaining a small part left. To ensure that the entire profile will be analysed, the same procedure is made from the opposite side. Consequently, \( 2N_s \) segments are obtained.

For each of the \( 2N_s \) segments, a polynomial function is fitted by least square to represent the local trend \( y_s(i) \). Then, the profile \( y(i) \) is “detrended” by subtracting the local trend \( y_s(i) \)

\[
F^2(t, s) = \frac{1}{s^2} \sum_{i=1}^{s} \{y[(v - 1)s + i] - y_s(i)\}^2,
\]

for \( t = 1, 2, \ldots, N_s \). The same must be done for the other half of the segments,

\[
F^2(t, s) = \frac{1}{s^2} \sum_{i=1}^{s} \{y[N - (v - N_s)s + i] - y_s(i)\}^2,
\]

for \( t = N_s + 1, N_s + 2, \ldots, 2N_s \). Finally, the average is taken over all segments

\[
F(s) = \left( \frac{1}{2N_s} \sum_{i=1}^{2N_s} [F^2(t, s)] \right)^{1/2}.
\]

Peng et al. (1994) used a linear fit to remove the trends, defining the default DFA polynomial order as 1. However, other polynomial orders can be used, as quadratic, cubic or higher. When it happens, the method is called DFA1, DFA2, DFA3, ..., according to the order engaged (Bunde et al., 2000). Depending on the fit, different tendencies can be removed, changing the results.

Then, one can show that typical data for \( F(s) \) have a linear behavior on double logarithmic plots (Peng et al., 1994; Ossadnik and Buldyrev, 1994). Then, the \( \alpha \) exponent can be obtained respecting the following power-law

\[
F(s) \sim s^\alpha.
\]

Fitting the log–log plot by least-square produces a straight line with slope \( \alpha \). If \( \alpha \approx 1/2 \) the signal is uncorrelated or there is only short-range correlations. On the other hand, if \( \alpha \neq 1/2 \) the signal has a long-range power-law correlation, i.e., there is no characteristic length for the correlations. Also, the \( \alpha \) exponent can be associated with the \( \beta \) exponent of the power spectrum technique through the following relation

\[
\beta \approx 2\alpha - 1.
\]

The PSD/DFA relation can be derived using the Wiener–Khinchin theorem (Heneghan and McDarbyee, 2000).
References