Shocks associated with the Kelvin–Helmholtz-resistive instability

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In this paper, a new type of shock associated with magnetic reconnection processes has been explored using a compressible magnetohydrodynamics simulation method. The simulations have shown that, when there are strong field-aligned shear flows at the two sides of a current sheet, the coupling mode between Kelvin–Helmholtz and resistive instabilities will appear; further, reflected shocks and incident shocks can be produced at both sides of the boundary layer. Both the reflected shocks and incident shocks are fast shocks, through which the magnetic field strength, density, and temperature all increase sharply, while the plasma velocity decreases steeply. It is expected that some inhomogeneous structures can be formed at plasma boundary layer regions due to the existence of fast field-aligned shear flow driven shocks. © 2000 American Institute of Physics.

I. INTRODUCTION

Magnetic reconnection processes play very important roles in solar and magnetospheric dynamics. In space environments, strong flow shear may exist, besides magnetic shear, e.g., in the solar atmosphere and at the magnetopause boundary layer. In these situations, the Kelvin–Helmholtz instability and resistive instability may interact with each other. Some researchers have explored the effects of shear flows on the resistive instability. It has been found that, for cases where there are weak shear flows, the resistive instability will be depressed by shear flows and/or fluid viscosity. However, when there are strong shear flows, the Kelvin–Helmholtz (KH) instability and resistive instability may interact and couple with each other so as to give rise to a new type of instability, which has been called vortex induced reconnection (VIR) or Kelvin–Helmholtz-resistive instability (KHRI). In this situation, the strong shear flows could destabilize the resistive instability and enhance the growth of magnetic reconnection. It has also been found that very large fluid viscosity could enhance the resistive instability when there was strong flow shear. Numerical investigations on KHRI have also revealed that KHRI only occurs in a limited range of shear flow velocities due to the compressibility of the plasmas, which is consistent with the observed results of magnetopause reconnection. KHRI has offered a mechanism for the fast magnetic energy release in space plasmas.

For some situations, various shocks may appear along the magnetic reconnection processes. In Petschek’s fast reconnection model, which was modified from the Sweet–Parker model, there existed a couple of slow shock pairs, so that two plasma jets carrying the reconnected magnetic field move away quickly from the X points at the ambient Alfvén speed. This kind of reconnection pattern has been found in the distant magnetotail. Further, developed the Sweet–Parker theory and Belle-Hamer et al. have explored the Sweet–Parker magnetic reconnection configuration when shear flows exist along with the boundary layer. In this situation, the shock structure becomes nonsymmetric. Some researchers have also studied the formation of shocks during the Kelvin–Helmholtz instability.

For situations with strong shear flows beside the current sheet, the KHRI process would occur. During this process, the current sheet becomes wavy, and the produced magnetic islands have an “S” shape. There is no doubt that a magnetic island would obstruct the strong shear flows, and their interaction would create shock waves beside the current sheet. The purpose of this exploration is to numerically study the shocks during KHRI processes.

II. SIMULATION AND RESULTS

We will use compressible magnetohydrodynamic (MHD) equations such as Eqs. (1)–(5) to simulate the nonlinear evolution of KHRI,

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0, \tag{1}
\]

\[
\frac{d \mathbf{V}}{dt} = -\frac{1}{2} \beta \mathbf{V} \cdot \nabla T + (\nabla \times \mathbf{B}) \times \mathbf{B} + \mu \nabla^2 \mathbf{V} + \frac{1}{3} \nabla \left( \nabla \cdot \mathbf{V} \right), \tag{2}
\]

\[
\frac{d T}{dt} = (\gamma - 1) \left[ -\rho T \mathbf{V} \cdot \nabla + \frac{\eta}{\beta} (\nabla \times \mathbf{B})^2 - \frac{2}{3} \mu (\nabla \cdot \mathbf{V})^2 + 2 \mu (\nabla \cdot \mathbf{V}) \mathbf{V} \right], \tag{3}
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}, \tag{4}
\]

\[
\nabla \cdot \mathbf{B} = 0. \tag{5}
\]
All physical quantities have been normalized by the mass density, temperature, magnetic field strength, and Alfvén speed in the ambient plasma far away from the current sheet. The tensor \( S \) is defined as \( S_{ij} = \frac{1}{2} (\partial_i V_j + \partial_j V_i) \). The adiabatic constant is \( \gamma = 5/3 \). \( \eta \) and \( \nu \) are the normalized resistivity and viscosity, respectively.

We have considered situations when the magnetic fields at the two sides of magnetopause, i.e., in the magnetosheath and magnetosphere, have the same strength. We investigate a two-dimensional slab configuration, letting the ambient magnetic field and shear flows be in the same direction. It is defined that the \( x \) axis is parallel to the ambient magnetic field, the \( z \) axis is perpendicular to the current sheet, and the \( y \) axis is along the direction of current. All quantities remain unchanged in the \( y \)-axis direction. We have adopted a moving coordinate system where the flows at the two sides of the magnetopause boundary layer have the same velocity, but are in opposite directions. Assume the ambient magnetic and shear flow fields to have the following forms:

\[
B(x, z) = \tanh(z) \hat{x},
\]

\[
V(x, z) = M_A \tanh(R_{BV} z) \hat{x},
\]

where, \( M_A \) is the Alfvén Mach number and also the normalized velocity (denoted as \( V_0 \)) of the shear flows, \( R_{BV} \) is the flow shear parameter, i.e., the ratio between the widths of magnetic and flow shear layers. The ambient temperature is assumed to be homogeneous in space and \( T = 1 \). Based on the balance condition between the thermal pressure and the magnetic pressure, the ambient plasma density \( \rho \) can be determined by \( \rho = 1 - (1 - B^2)/\beta \).

We let the disturbances be along the ambient magnetic and flow field, and the wave vector \( \kappa \) of the disturbances is in the \( x \) direction. From Eqs. (6) and (7), the equilibrium magnetic flux function \( \psi_0 \) and stream function \( \phi_0 \) may be expressed as

\[
\psi_0(x, z) = \ln(c h(z)),
\]

\[
\phi_0(x, z) = \frac{M_A}{R_{BV}} \ln(c h(R_{BV} z)).
\]

At the beginning, we only give the KHRI type of disturbances to the magnetic field and flow field. We can add the Alfvén fluctuation caused by the KH instability and the concentric magnetic island and fluid vortex disturbances associated with the resistive instability to the equilibrium magnetic flux function and stream function so as to get the initial disturbed fields of KHRI as the following:

\[
\psi(x, z) = \ln(c h(z - z_m \exp(-\kappa z^2)\sin \kappa x)) + A_m \exp(-\kappa z^2) \cos \kappa x,
\]

\[
\phi(x, z) = \frac{M_A}{R_{BV}} (c h(R_{BV} z - z_m \exp(-\kappa z^2)\sin \kappa x)) + F_m \exp(-\kappa z^2) \cos \kappa x,
\]

where \( \kappa \) is the dimensionless wave number of the disturbance, and \( z_m \) is the maximum deviation of the magnetic field lines and streamlines from the equilibrium field configuration caused by the KH instability. \( A_m \) and \( F_m \) are the maximum disturbances of the magnetic flux function and stream function in the magnetic islands and fluid vortices.

The initial disturbed magnetic field and fluid velocity field can be obtained from the relation formulas \( \mathbf{B} = \hat{y} \times \nabla \psi \) and \( \mathbf{V} = \hat{y} \times \nabla \phi \).

In the simulations, the boundaries in the \( z \) direction at \( z = \pm 7 \) (for the first case) or \( z = \pm 8 \) (for the first case) are free and satisfy the Sommerfeld condition, while the boundaries in the \( x \) direction are chosen as periodic. We have used a Runge–Kutta scheme that has fourth-order accuracy in time and second-order accuracy centered differences in space. The grid numbers in the \( z \) and \( x \) directions are 700 \times 600 (for the first case) or 600 \times 600 (for the second case). In our simulations, \( 0.01 \leq \eta > \nu \leq 0.001 \), the viscous boundary layer is thinner than the resistive boundary layer and has a half width of about \( \varepsilon = (\nu/\kappa)^{1/3} \geq 0.13^{1/3} \) where \( \kappa = 0.5 \). Because the total lengths in the \( z \) and \( x \) directions are 14 or 16 and \( 2\pi/\kappa = 12.5 \), respectively, there should be ten grid points or more in the resistive or viscous boundary layer, thus various dissipation processes can be properly simulated.

Figure 1 illustrates the typical evolution of various physical fields in the process of KHRI, which has shown that the shock waves could be produced by KHRI. In the case of Fig. 1, the wave number \( \kappa = 0.50 \), dimensionless resistivity \( \eta = 0.01 \), dimensionless viscosity \( \nu = 0.001 \), plasma beta \( \beta = 0.4 \), shear flow Mach number \( M_A = 1.2 \), and flow shear strength \( R_{BV} = 1.2 \). With the time increasing, the KHRI process gradually develops, the originally straight current sheet becomes wavier and wavier, and concentric magnetic island and fluid vortex begin to appear at the current sheet. The X points have the maximum values of the current density, vorticity, mass density, and temperature due to the compression caused by the opposed flows at the two sides of the X point. It can be seen that, during the evolution of the KHRI process, the shocks are being formed on both sides of the current sheet, which is indicated by the steep changes of the directions and strengths of the magnetic field and flow velocity, the very large current density and vorticity, and also the high gradients of the mass density and temperature. At each side of the current sheet, there is one shock starting from the concave part of the current sheet and another starting from the convex part. The former is caused by the reflection of the shear flow and magnetic field from the current sheet, and the flow velocity and magnetic field at the downstream deviate away from the current sheet. The latter is caused by the incidence of the shear flow and magnetic field to the current sheet, and the flow velocity and magnetic field at the downstream deviate toward the current sheet. Therefore, they are called reflected shock and incident shock, respectively. The reflected shock inclines toward the downstream while the incident shock leans toward the upstream. The shocks end outside the current sheet and also outside the magnetic island and they cannot stretch into the boundary layer due to the rather small speed of the flows there.

As the shocks are weak, the angle between the reflected shock front and the \( x \) axis is \( \alpha \approx 63.4^\circ \), while the angle between the incident shock and the \( x \) axis is about \( 121^\circ \). When
the shocks become stronger at the later stage, these angles change more or less.

We now investigate the reflected shock and incident shock associated with KHRI processes in detail. Figure 2 illustrates the change of various physical quantities along the line \(z = 4\) at the time \(45\tau_A\) for the above-mentioned case. The sharp peaks of current density and vorticity at \(x = 6.52\) and \(x = 10.44\) mark the locations of the reflected shock and incident shock. It can be seen from Fig. 2 that, through both the reflected shock and incident shock, the magnetic field direction deviates away from the shock normal, and the magnetic field strength, mass density, and temperature have all been enhanced considerably, while the plasma velocity has been reduced sharply. Thus the two shocks are all fast shocks.

FIG. 1. The evolution of magnetic field, flow field, current density, vorticity, mass density, and temperature during the formation of the shocks in the KHRI process. The parameters are \(\kappa = 0.50\), \(\eta = 0.01\), \(\nu = 0.001\), \(\beta = 0.4\), \(V_0 = 1.2\), and \(R_{BV} = 1.2\). The plots of current density and vorticity have 20 contour levels, while the plots of mass density and temperature have 30 contour levels.
We may check the Rankine–Hugoniot relations of the reflected shock and incident shock for this case, i.e., the conservation of mass flux, magnetic flux, tangential electric field, momentum flux, and energy flux across the shock layers. Here we define the mass flux as $F_m = \rho V_n$, the magnetic normal component $B_n$, tangential electric field $E_t = B_n V_t$, normal momentum flux $P_n = p + \rho V_n^2 + (B_t^2 - B_n^2)/2$, tangential momentum flux $P_t = \rho V_t V_n - B_t B_n$, and energy flux $W = pV_n + \rho V_n^2 \left( \frac{\gamma}{\gamma - 1} \frac{p}{\rho} + \frac{1}{2} V^2 \right) + B^2 V_t - B_n (V \cdot B)$, where $V_n$ and $V_t$ are the components of flow velocity normal and tangential to the relevant shocks, respectively, and $B_t$ is the component of magnetic field tangential to the relevant shocks. All these physical quantities are in normalized units. Table I shows the calculated conservation quantities at the upstream and downstream of the deflection shock and incident shock along $z=4$ and at time $45\tau_A$ and also the relevant error. The error is defined as $|f_2 - f_1|/|f_1|$, where $f_1$ and $f_2$ are the conservation quantities at the upstream and downstream of the shocks. Table I indicates that the Rankine–Hugoniot relations of the reflected shock and incident shock are satisfied well, except for the tangential electric field $E_t$ and energy flux $W$ for the reflected shock. The reason for this deviation is that the shocks at the time $45\tau_A$ are not stationary and are still growing, and the tangential magnetic flux and the plasma energy inside the shock layers are gradually increasing, so that $E_t > E_t^1$ and $W_t > W_t^1$. Therefore we can believe that the simulation results have physically revealed the existence of shocks during KHRI processes.

Figure 2 also shows that, in the downstream of both shocks, the magnetic field strength, density, and temperature being to decrease slowly, and the plasma velocity increases gradually. With the time increasing, both shock fronts grow thinner, and their half widths may decrease up to about 0.2 or a fifth of that of the initial current sheet as the saturation of KHRI arrives.

We have also simulated many other cases with different parameters. Figure 3 shows another case when the plasma beta $\beta = 1$, $M_A = 1.4$, $R_{BV} = 1.6$, and the other parameters are the same as the above-mentioned first case. Figure 3 only illustrates the various physical fields when KHRI saturates at time $= 63\tau_A$. Similar to the first case, both the reflected shock and incident shock are produced in this situation. Figure 4 demonstrates the variations of the different physical quantities along the line $x=4.8$ when the KHRI saturates. The reflected shock and incident shock is at about $x=1$ and $x=7.2$. Crossing both the reflected shock and incident shock, the plasma flow velocity drops, and the magnetic field

<table>
<thead>
<tr>
<th></th>
<th>$F_m$</th>
<th>$B_n$</th>
<th>$E_t$</th>
<th>$P_n$</th>
<th>$P_t$</th>
<th>$W$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upstream of deflection shock</td>
<td>0.9810</td>
<td>0.8273</td>
<td>-0.0190</td>
<td>1.0002</td>
<td>0.1434</td>
<td>1.3388</td>
</tr>
<tr>
<td>Downstream of deflection shock</td>
<td>0.9696</td>
<td>0.8202</td>
<td>-0.0146</td>
<td>0.9992</td>
<td>0.1455</td>
<td>1.1263</td>
</tr>
<tr>
<td>Error</td>
<td>0.011</td>
<td>0.0086</td>
<td>0.23</td>
<td>0.0009</td>
<td>0.014</td>
<td>0.16</td>
</tr>
<tr>
<td>Upstream of incident shock</td>
<td>1.119</td>
<td>0.9428</td>
<td>0.0242</td>
<td>1.0702</td>
<td>-0.1176</td>
<td>1.3135</td>
</tr>
<tr>
<td>Downstream of incident shock</td>
<td>1.055</td>
<td>0.8872</td>
<td>0.0234</td>
<td>1.0492</td>
<td>-0.1257</td>
<td>1.2129</td>
</tr>
<tr>
<td>Error</td>
<td>0.058</td>
<td>0.059</td>
<td>0.047</td>
<td>0.020</td>
<td>0.069</td>
<td>0.077</td>
</tr>
</tbody>
</table>

FIG. 2. The variations of magnetic field strength, velocity, current density, vorticity, mass density, and temperature by $x$ along the line $z=4$ at $45\tau_A$ for the case of Fig. 1.
of the values of the magnetic field and flow velocity will bring about the inaccuracy of the first effective digit of the value of the electric field. Therefore, some existing disturbances at the fast shock fronts may lead to small deviations of the mass flux, magnetic flux, momentum flux, and energy flux and large deviations of the tangential electric field from the Rankine–Hugoniot relations.

It is noticed that there are periodic structures at the reflected shock and incident shock layers, as illustrated from the current density and vorticity distributions in Fig. 3. Actually, this is a common feature of the fast shocks arising from KHRI, especially as the shock fronts grow very thin. These periodic structures are some disturbances that are possibly produced by a certain instability occurring at the fast shock fronts with very large current density and vorticity.

It has been found that, in general, if KHRI occurs as the shear flows have a velocity near the fast magnetosonic speed, one pair of reflected shock and incident shock at each side of the current sheet will appear. These shocks are all fast shocks and the Rankine–Hugoniot relations are basically satisfied. However, it has also been revealed that high plasma beta does not favor the development of KHRI and the formation of fast shocks. We have calculated a number of situations with very high plasma beta. For example, when $\beta = 4$, $V_0 = 2.3$ and other parameters are the same as in the above two cases, the magnetic reconnection is very weak, and there is no shock formed. The MHD simulations have shown that, the higher the plasma beta, the weaker the KHRI and the produced fast shocks. The reason should be that, for very high plasma beta situations, the magnetic fields are too weak for the KHRI to evolve and the fast shock cannot be produced. It is indicated that the shock formation during KHRI is a magnetohydrodynamic phenomenon.

We may calculate the angle between the shock front and current sheet based on theoretical considerations. We can regard that the wavy current sheet obstructs the fast flow and stimulates fast magnetosonic waves propagating outward so as to cause the formation of the reflected shock front. The phase speed $V_{ph}$ of the fast magnetosonic waves satisfies

$$2V^2_{ph} = (V^2_A + C^2_S) + \sqrt{(V^2_A + C^2_S)^2 - 4V^2_A C^2_S \cos^2 \theta},$$

where $\theta$ is the angle the fast wave traveling direction makes with the ambient magnetic field, $V_A$ is the Alfvén speed, and $C_S$ the speed of sound. In the normalized units, $V_A = 1$, $C_S = (\gamma/\beta)^{1/2}$, $\gamma = 5/3$. The curve in Fig. 5 shows the variation of the phase speed of fast magnetosonic waves with the propagation angle $\theta$. Figure 5 demonstrates the formation of shocks in the fast flow with velocity $V_0$, i.e., $M_A > 1$ with very high plasma beta situations. The shock front is tangent to the phase speed curve of the magnetosonic waves. It is difficult to get the explicit formulation for the slope angle of the shock front. Here we use Newton’s method to numerically calculate the slope angle of the shock front. The numerical calculation showed that for the first case, as the plasma $\beta = 0.4$ and shear flow velocity $M_A = 1.2$, the angle between the shock front and magnetic field is $\alpha \approx 62.0^\circ$; for the second case with $\beta = 1.0$ and $M_A = 1.4$, the angle between the shock front and magnetic field is $\alpha \approx 63.5^\circ$.

strength and thermal pressure rise abruptly. Both the reflected shock and incident shock are fast shocks. The incident shock is very sharp and its half width is only about 0.2, while the half width of the reflected shock is about 0.4. The angle between the reflected shock front and $x$ axis is $\alpha = 63.5^\circ$, and between the incident shock and the $x$ axis is about $117.7^\circ$. The Rankine–Hugoniot relations for the reflected shock and incident shock at saturation in this case have been checked as in Table II. The conservation of the mass flux, magnetic flux, momentum flux, and energy flux across the shock fronts are all satisfied well, with errors less than 0.066, except for that of the tangential electric field. The errors of the tangential electric field are about 0.63. The reason for this is that the value of electric field, i.e., $E = -V \times B$, is very small and of the order of 0.01 (note that the magnetic field and flow are almost parallel with each other), while the mass flux, magnetic flux, momentum flux, and energy flux all have the order of 1. So that if the errors of the magnetic field and flow velocity are about 0.01, the correspondingly deduced quantity electric field will have an error of about 1 because the inaccuracy at the third effective digits
front and magnetic field is $\alpha=60.0^\circ$. Therefore, the above-mentioned MHD simulation results are consistent with calculations here based on theoretical analysis.

At each side of the current sheet, one reflected shock and one incident shock combine with each other to facilitate the growth of KHRI. As the shear flows run through the two reflected shocks, the magnetic field strength and thermal pressure increase and the plasma velocity decreases steeply, so that at the $X$ point the current sheet will be thinned and current density strengthened considerably. This is rather advantageous for the development of the magnetic reconnection. The incident shocks make the magnetic field and flow field bend to the current sheet so as to form the upstream of the deflection shocks.

If we transfer the calculation coordinate system into the actual coordinate system in which the plasmas inside the magnetosphere have zero speed, then the velocity of the magnetosheath flow should be $2M_A$, and the shocks in the magnetosheath are moving at a speed of $M_A$ that is half of the sheath flow speed.

### III. SUMMARY AND DISCUSSION

In this paper, we have numerically explored the formation of shocks due to the interaction between fast shear flows and the KHRI process.

The main results obtained are summarized as follows.

1. In the KHRI processes, the current sheet gradually forms an “S” shape structure due to the KH instability. The super Alfvén flows interact with the wavy current sheet so as to produce shocks.

2. The shocks may occur on both sides of the current sheet. At each side, there is a reflected shock started from the

<table>
<thead>
<tr>
<th>TABLE II. The calculation of Rankine–Hugoniot relations for the case of Figs. 3 and 4.</th>
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<tbody>
<tr>
<td>$F_m$</td>
</tr>
<tr>
<td>-----------------</td>
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<tr>
<td>Upstream deflection shock</td>
</tr>
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<td>Error</td>
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</table>
concave part of current sheet and an incident shock started from the convex part.

(3) Both the reflected shock and incident shock are fast shocks. Through both of them, the magnetic field strength, density, and temperature all increase sharply, while the plasma velocity decreases steeply.

(4) The shocks do not extend into the boundary layer.

(5) When the fast shocks become very thin, some small-scale periodic structures appear in the shock front layers.

(6) Very high plasma beta does not favor the formation of the fast shocks associated with KHRI.

(7) The reflected shocks and incident shocks formed at the two side of the current sheet are very advantageous for the evolution of KHRI processes.

The shocks associated with KHRI are different from those in Petschek’s fast reconnection model, which are slow shocks. The shocks associated with KHRI are caused by fast shear flows parallel to the magnetic field, while the slow shocks in Petschek’s fast reconnection process are driven by flows perpendicular to the magnetic field and running toward the neutral sheet.

This investigation has indicated that, due to the existence of KHRI shocks, some inhomogeneous structures can be formed at plasma boundary layer regions.

We expect that, after the successful launch of CLUSTER II into orbit, the instruments on board will make three-dimensional and small-scale structure measurements of this kind of shocks associated with KHRI.

The interaction between shocks and boundary layers in magnetized plasmas is a complicated problem that needs more efforts on theoretical analyses and numerical investigations.

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7 C. Shen and Z. X. Liu, in Magnetospheric Research with Advanced Technique, COSPAR Colloquium Vol. 6 (Elsevier Science, Oxford, 1997).